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THE UNAIDED EYE
PART II

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THE previous lecture was confined to the dioptric mechanism of the eye; the means by which the images of external objects are formed upon the retina; and the mechanism which controls the quantity of light that enters the eye. This lecture will deal particularly with the sensory elements, the organs which appreciate the form, brightness, and colour of the image and transmit these sensations to the brain.

The eyeball is a semi-rigid and, so far as the rear portion is concerned, an approximately spherical body. Its wall may be divided into two distinct layers, an outer, comparatively thick, white, nearly opaque, fibrous coat, called the sclerotic, and an inner, soft, extensible, brownish-coloured, and more complex, thinner layer called the choroid membrane. Upon the inner surface of the choroid there lies the retina, which receives the light that enters the eye.

To the sclerotic, which forms the true wall or skeleton of the eye, there are attached externally the motor muscles. Its substance is pierced at the base towards the nasal side by the optic nerve which passes through a tube or sheath continuous with the sclerotic of the eye at the one end and the dura mater of the brain at the other.

The choroid, which is only lightly attached to the inner surface of the sclerotic, may again be roughly divided into three layers:

(1) An outer layer, adjacent to the sclerotic, containing a number of black pigmented cells, to which the brownish appearance is due. Probably the function of these cells is to absorb the greater part of the light that passes through the retina.

(2) An intermediate layer of tissue traversed by a network of arteries and veins and containing more widely dispersed pigmented cells. These arteries and veins

have been used to demonstrate the location of the visual perceiving organs in front of the choroid.

(3) A thin internal hyaline membrane, highly transparent and elastic, upon which the pigmentum epithelium, the basal layer of the retina, rests. The choroid extends as an inner sheath within the dural sheath around the optic nerve and is continuous with the pia mater of the brain. By Mariotte and other well qualified observers, it was contended that the choroid and not the retina was the seat of visual perception. This view has long since been abandoned. It is sufficient here to state that, while the retina ends at the ora serrata, beyond which there is no light perception, the choroid extends as far as the ciliary muscle.

The retina covers the whole of the inner surface of the eye, as far as the ora serrata, where it is abruptly terminated.

If an equatorial section of the eye is made and the front portion removed, two important spots on the retina may at once be observed. One of these—a circular spot—situated towards the nasal side of the axis, is the place where the optic nerve pierces the sclerotic and choroid and spreads its fibres over the inner surface. It is commonly called the blind spot, as it is devoid of all visual sensation. The main artery and vein which supply blood to the retina pass into the eye through the centre of the blind spot and spread as a fine network through the surface layers of all parts of the retina with the exception of the macula lutea. The other spot, which is oval-shaped, lies nearly on the axis of the eye, close to the blind spot. It is called the macula lutea, or yellow spot, on account of its yellowish colour. The image of an object observed directly by the eye falls upon the centre—the fovea centralis—of this spot. According to Gullstrand, however, the yellow tint from which the name macula lutea is derived is a post-mortem effect which is not present in the freshly excised eye. If this statement were correct, it would be necessary to revise many of the conclusions of Clerk Maxwell and other noted authorities which are based on the existence of a yellow pigment throughout the substance of the macular area in the living eye.

At the section where the optic nerve pierces the retina it is considerably constricted, thus reducing the ineffective blind portion to a diameter of about 2 mm. The edge of the macula lutea, which has an approximate width of about 2 to 3 mm. horizontally and about 1·0 mm. vertically, approaches very closely to the retinal edge of the blind spot. The fovea centralis, which is also slightly oval, has an approximate diameter of 0·25 mm. and subtends an angle of about 1°, the average value for the macula itself being 8°.

The retina is a most remarkable series of layers of which there are no less than ten that are well defined, but whose functions are very largely hypothetical. Fig. 1 is a section through the several layers for which I am indebted to Dr Maitland Ramsay. The direction of the incident light is indicated by the arrow and in order to reach the ninth layer which comprises the peripheral sensory elements, it will be seen that the light must pass through the intervening eight layers. These layers are actually transparent, the opacity indicated in the photomicrograph being due to the method of staining the specimen.

Considering the complexity of the details, it is reasonable to assume that there are small differences of refractive index that will deviate individual rays and possibly reduce the sharpness of the image on the ninth layer, but, on the other hand, the total thickness of the layers is actually very small, being not more than about 0·4 mm. at the thickest part, in the neighbourhood of the blind spot, and considerably less in the macula lutea itself, and particularly in the fovea centralis. Thus, assuming the thickness of the intervening layers to be 0·1 mm. near the position of direct vision and that a ray is deviated by ± 1 minute of angle, then the confusion of the image from this cause would be only 1/30 of the minimum con-

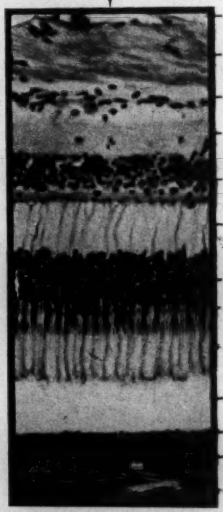

Fig. 1. Section of retina.

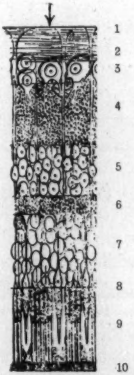

Fig. 2.

fusion attributable to diffraction, assuming the maximum diameter of the pupil to be 6 mm.

Diagrams of the retina usually suggest a detailed histological knowledge which it is difficult to appreciate from an examination of actual sections, however well produced. But a considerable amount of the detail has been traced by the adoption of various methods of staining and particularly by the application of Golgi's silver process. The functions of most of the parts are still, however, very obscure.

Fig. 2 is a diagrammatic section, extracted from Helmholtz's *Physiologische Optik*, the layers of which are numbered similarly to those of the actual section, Fig. 1. The arrow indicates the direction of the light approaching the retina. The layers are as follows:

(1) The membrana limitans interna, which is a very thin, transparent layer covering the retina proper and upon which the hyaloid membrane enclosing the vitreous humour rests.

(2) The layer of optic nerve fibres which radiate from the blind spot in all directions and finally pass normally through the intervening layers towards the corresponding rods or cones, except at the macula lutea, where they are so inclined as to reduce the thickness of the layers over the layer of cones.

At the blind spot, where the optic nerve enters the eye, the layer is thickest, and, as is to be expected, the thickness decreases towards the extreme boundary of the retina, due to the spreading of the nerves over a greater surface.

(3) The inner ganglion cell layer which comprises a number of well defined cells. Near the macula lutea the layers of cells increase in thickness but in the fovea centralis the thickness is again greatly diminished, thus reducing the depth of the cone layer below the first surface layer. These ganglion cells appear to be closely associated with the individual nerve fibres.

(4) The inner molecular layer, comprising an ill-defined mass of small neurones interwoven with minute nerve branches.

(5) The inner granular layer, comprising several layers of nerve cells somewhat similar to, but considerably smaller than, those of the third or ganglion layer.

(6) The outer molecular layer is described by Helmholtz as having a very mysterious structure and but little is yet known regarding it. Certain so-called horizontal cells are supposed to interconnect or associate a number of rods with one another.

(7) The outer granular layer resembles the fifth inner granular layer.

(8) An extremely thin layer called the membrana limitans externa which may serve to retain the rods and cones of the ninth layer in their relative positions, or which may serve as a diaphragm retaining the liquid surrounding the rods and cones, as is suggested by Dr Edridge-Green.

(9) The layer of rods and cones which are generally regarded as the true peripheral organs of visual perception. It will be considered in greater detail later.

(10) The layer of pigment cells resting upon the choroid previously described. It consists of well defined hexagonal cells, as indicated in the photomicrograph, Fig. 3.

From the cells a number of processes extend between the rods and cones, separating them from one another. A black pigment within the cells possibly serves to absorb light that passes the ninth layer, but this is probably not its sole function, because under the prolonged action of light the black pigment moves forward as far as the membrana limitans externa in the case of the lower animals and actually leaves the cells entirely. In the case of man, however, this light effect is comparatively slight.

The epithelium cells of the tenth layer appear to have some connection with the production of the visual purple in the rods but the black pigment itself appears to play no direct part in its formation, since visual purple is said to be found in the eyes of albinos, in which the pigment of the epithelial cells is lacking.

Although very little is known regarding the true functions of these various elements of the retina, certain relationships between the parts have been traced and there is a considerable amount of evidence in support of some of the conclusions arrived at, such, for example, as the opinion that the ninth layer of rods and cones is the seat of visual sensations which are translated by the brain as the sensations of form, intensity, and colour.

In Fig. 4, which is a diagrammatic section of the retina, the arrangement of some of the simpler elements associated with the rods and cones is indicated. There are, however, various other complex details which are fully described in Luciani's *Human Physiology*, vol. IV, "The Sensory Organs." Light which enters the eye impinges on the retina in the direction of the arrow, Fig. 4, and after traversing the layers 1 to 8, it reaches and excites the ninth layer of rods and cones. These elements act as transformers. They convert the energy of the incident light vibrations into a form capable of exciting the nerves which are quite irresponsive

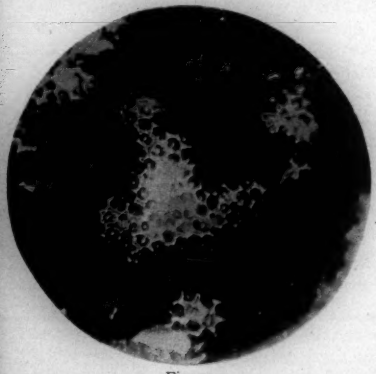

Fig. 3.

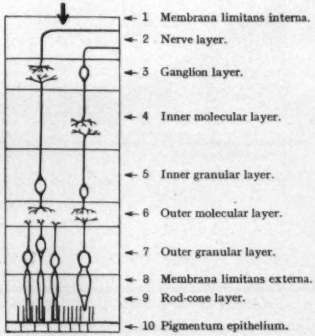

Fig. 4.

to the light waves themselves. This transformed impulse is transmitted in the case of the cones through a nerve cell of the seventh layer to the ramifications of the nerve fibre in the sixth or outer molecular layer. Similar ramifications of a corresponding nerve fibre appear to collect the impulse and transmit it through a nerve cell of the fifth layer to the branches of the nerve fibre in the fourth or inner molecular layer where the impulse is again collected and transmitted through a ganglion cell of the third layer, a nerve fibre of the second layer, and the optic nerve to the brain.

One nerve fibre is generally allocated to each cone of the fovea centralis where, indeed, only cones exist. In other parts of the retina two or more even appear to be associated with one fibre. This is particularly the case where the rods are concerned. As is indicated in Fig. 4, several rod fibres are grouped in the outer molecular layer by one nerve fibre which carries the sensation to the brain as before. As

there are about three million cones and about five or six times as many rods, that is a total of very approximately eighteen million elements, and only one million nerve fibres in the optic nerve which leaves the eye, some such grouping as is described above appears to be a reasonable assumption.

It will be observed that, before reaching the ninth layer, the light has to traverse the second layer of nerves through which the visual sensations are being transmitted from the excited rods and cones to the brain. If these nerves were visually responsive to light itself, it would be difficult to understand how the rod and cone impulses could be separated out with the distinctness characteristic of acute vision. It would appear, however, that, although the nerves can be excited by chemical, electrical and mechanical means, they do not respond to light vibrations. Thus, the light acts on the nerves only through the intermediary of the rods and cones. If a beam of light is concentrated on the blind spot where only nerves and no rods or cones exist, there is no resultant visual effect produced.

After the nerves leave the blind spot, they lose their medullary sheath and spread as a fine network over the whole of the retina, but at the fovea centralis where the layers in front of the rods and cones are greatly reduced in thickness, and in some cases absent, the nerves pass obliquely to the cones, as is indicated in Fig. 5, which is extracted from Helmholtz's *Physiologische Optik*. The artery and vein which supply the blood to the retina enter through the centre of the blind spot and spread as very fine walled vessels through the whole of the retinal surface except the macula lutea. These blood vessels are not wholly transparent. When suitably illuminated they cast shadows on the underlying layers and it is this fact that was utilised to demonstrate the sensory function of the rods and cones. If a bright image is focussed on the exposed exterior part of the sclerotic, a portion of the small beam of light will fall on the retina and the shadow of the blood vessels will be cast directly on the sensory part of the retina, if the vessels lie in front of it. Thus the brain will see the shadow of the blood vessel on a small illuminated background. If, when the light is moved, the shadow moves in the same direction, as is the case, the blood vessel casting the shadow must lie in front of the sensory element. The depth of the sensory layer may even be calculated very approximately from the amount of the displacements. From this and other considerations there seems to be little reason to doubt that it is the layer of rods and cones that is the true seat of visual perception.

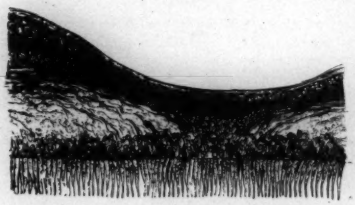

Fig. 5.

Before attempting to discuss a theory of vision, it is necessary to consider the capabilities of the eye as regards the appreciation of light, form, and colour, because in view of the indefiniteness of the available histological information it is only upon such data that a satisfactory theory of vision can be based.

So far as the appreciation of the intensity of light is concerned, very much depends upon the conditions of the experiment. For example, the continuous field of view of a telescope may appear uniformly illuminated and yet there may be a difference of 100 per cent. in the intensity between the centre and the edge. Designers of optical instruments very frequently make use of this fact, providing only half light at the periphery. Further reduction of the light beyond this amount may, however, be quickly detected. When, however, two fields are brought into sharp contrast, being separated from each other by a fine line, an intensity difference between them of three per cent. or possibly less may be distinguished with certainty. For example, if the illuminations of the two fields are the same and a glass plate is inserted in one beam, the four per cent. loss of light thus produced will be readily seen. But if a one per cent. difference which cannot be detected with certainty is produced by means for example of a diaphragm in one beam and if the diaphragm is moved in and out of the beam, the difference may then be observable as a flicker appearance. The more sudden the transition, the more evident will be the change. The increased accuracy obtainable by flicker photometer methods is well known.

For the tests, the results of which are indicated in Fig. 6, a coincidence range-finder, the fields of which are contiguous and separated by an extremely fine line, was selected. The entrance pupil of the one optical system was kept constant while that of the other was varied by the small amounts in question. An iris diaphragm is not a suitable means of varying the aperture, owing to the difficulty of measuring the diameter with sufficient accuracy. Instead of the iris diaphragm, a series of simple metal diaphragms was prepared having apertures which varied in area by steps of 0·5 per cent. The observer did not know in which beam the constant diaphragm was placed, nor the order in which the variable diaphragms were inserted. Observations were made only after the diaphragms had been changed, not during the process.

It is necessary to express such photometric results on a frequency basis, for, although the eye is able to detect a three per cent. difference with absolute certainty, it may be able to detect a much smaller percentage difference with some certainty. Thus in Fig. 6, the ordinates represent the percentage of correct estimations. The number 50, therefore, represents the absolute uncertainty line, as the observer was as often wrong as right. The 100 per cent. ordinate represents absolute certainty, as the observer was correct every time. Abscissae represent the percentage differences in the brightnesses of the two fields and as the standard diaphragm was placed in either beam the curve is double-sided. It should be observed how closely the middle point of the curve falls on the zero difference line, thus indicating a very perfect matching of the two fields in the first instance. At the retina the angular field covered was 42° and the pupil diameter was about 2 mm.

The three curves show the results obtained when using as the illuminating source the light of a 200 c.p. lamp, a 40 c.p. lamp, and a 16 c.p. lamp respectively, reflected from a white surface. Between the 40 and 16 c.p. curves there is here practically no difference, and only a very slight intermediate difference in the case

of the 200 c.p. source. For all three illuminations the eye was able to appreciate with absolute certainty a 3 per cent. variation of the brightness of the two contiguous fields of the optical instrument. For a 1 per cent. difference the percentage of correct judgments was 75.

In the above test a considerable area covering about 42° of the retina was involved. It was thought desirable to examine in the same way small areas on the fovea centralis. For this purpose simpler direct vision apparatus was employed. It consisted of two semi-circular fields formed by light reflected from uniform

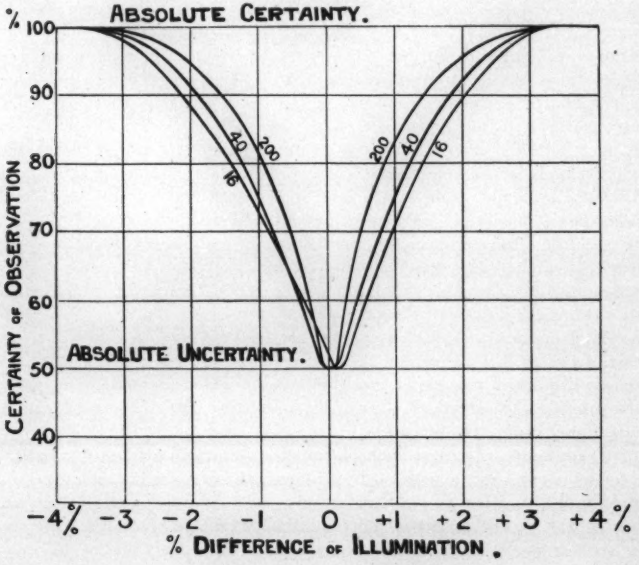

Fig. 6.

white surfaces, the fields being contiguous about a vertical diameter and situated at a distance of 100 inches from the observer's eye. The illumination of one of the fields was fixed: the other was varied by displacing the source along a scale. No stray light from the one field could reach the other and of course all stray light was prevented from reaching the observer's eye. A clearly defined image free from any star appearance was obtained by the use throughout of a millimetre diaphragm at the eye, as described in the first lecture. The vertical line of separation was clearly defined in all the experiments but in the case of the very small fields there was an appreciable broadening of the separating line, possibly attri-

butable to the complex systems of diffraction bands formed by such an arrangement of apertures.

The results are indicated in Fig. 7, the arrangement of which is the same as that of the previous figure, with the exception that the illuminations are represented upon a different scale of abscissae and that only the right halves of the curves are indicated. In all cases the vision was direct. When the area subtends 172 minutes, a 2½ per cent. difference is distinguishable with certainty. It will be recollected that in the previous experiment when using an optical instrument, a 3 per cent. difference was distinguishable with certainty when the retinal area illuminated was large. Although the conditions were very different in the two sets of experi-

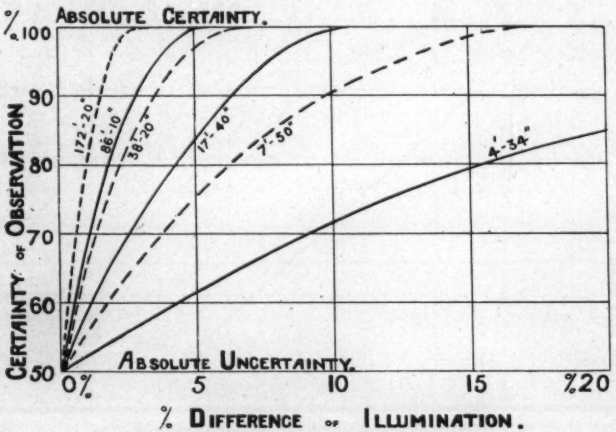

Fig. 7.

ments, it is probable that the results for the large 42° area and the smaller 2½° area are practically the same, as will be more clearly demonstrated later. When the angular area is 86 minutes, the percentage is about 5; when the angle is about 38 minutes the percentage is about 7; and when the angular area is reduced to about 8 minutes, an 18 per cent. difference can just be detected with certainty.

It is quite evident that when the retinal area subtended approaches one degree there is an increasingly rapid reduction in the sensitivity of the eye as regards light intensity. This is more clearly expressed in Fig. 8, which includes the previous results in addition to other series. It will be seen that the observations fall upon a very fair curve. The black curve of the diagram is plotted with respect to the angles subtended by the various holes as ordinates and the absolute certainty

percentage differences in illumination as abscissae. This curve very clearly shows that, whereas large retinal areas down to those subtending about one degree are capable of appreciating comparatively small changes in illumination, the central parts of the fovea centralis are remarkably insensitive. When the angular area is about four or five minutes, in the case of direct vision very large differences in brightness cannot be detected. This may conceivably be due to diffraction effects tending to confuse the two very small images, but on the other hand the line of separation between the two fields, although slightly broadened, appeared well defined and the form was clearly recognisable.

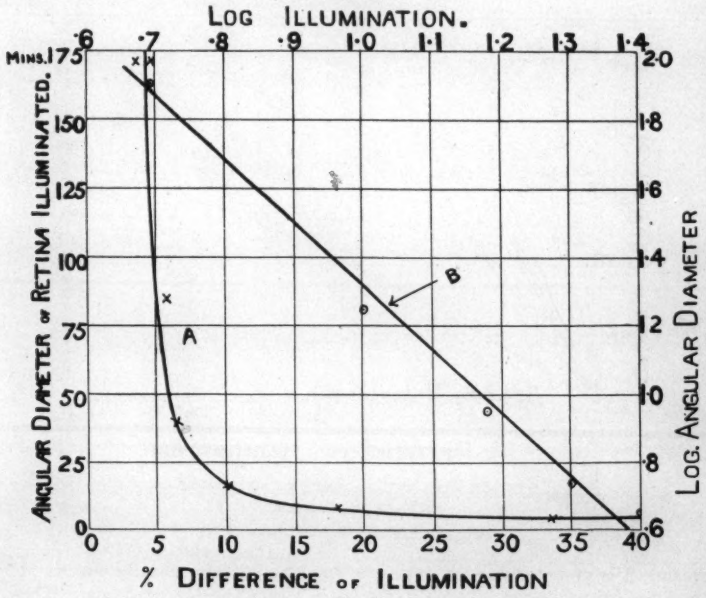

Fig. 8.

It would be interesting to know if the curve bears any definite relationship to the ratio of the rods and cones, the former being in the majority in the more peripheral parts of the retina and entirely absent over the central part of the macula lutea containing the fovea centralis. If this were the case, it would suggest an important difference between the functions of these elements. If, however, the adult rod-free area subtends an angle of three degrees, as is stated by Koster, the insensitiveness in question cannot be attributed to the absence of rods. It suggests the existence of a 4 or 5 minute foveola characterised by a distinct change in the

functions of the cones or the possible existence of some new feature. This figure of 4 to 5 minutes occurs frequently in a large variety of direct vision experiments. When the data in Fig. 8 are plotted logarithmically, as indicated in the red curve *B*, it will be seen that the greater portion reduces to a straight line. For this intermediate portion, if *B* is the percentage difference in brightness that can just be detected with certainty and *d* is the angular diameter of the retinal area illuminated under direct vision, then $B = \frac{1}{d^{0.52}}$, that is, *B* approximately varies inversely as the $\sqrt{d}$.

Similar curves have been obtained for light of various colours. A typical set for a hole subtending 7′ 50″ illuminated by white, yellow, red, and green light is

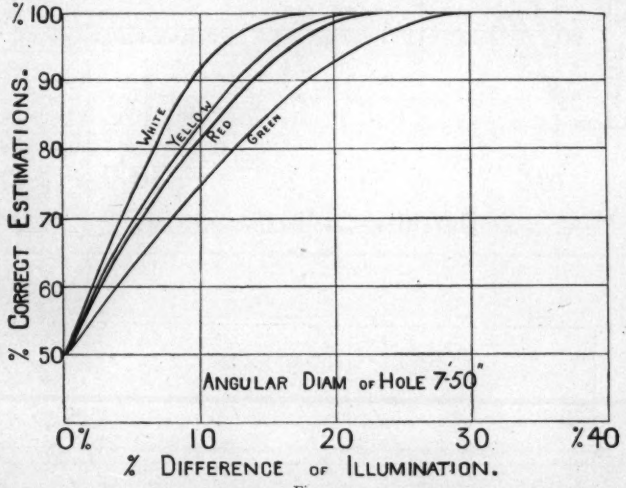

Fig. 9.

illustrated in Fig. 9. The illuminating source was a "Pointolite" lamp of 400 c.p., the light of which passed through a filter of the appropriate colour. It is possible that part of the differences, which are not very great, may be attributable to differences in the intensity of the light rather than to changes of the colour alone.

Similar curves have also been obtained for the same diameter hole, namely 7′ 50″, using coloured light of three intensities. Thus in Fig. 10 there are indicated three curves of different intensities for each of the colours, green, red, and yellow, when using illuminating sources of 400 c.p., 100 c.p., and 16 c.p. In the case of the green light the three curves are arranged in the order of the intensities, the sensitivity being least in the case of the lowest illumination, but the order is not the same for the red and yellow light, the order being actually the reverse in the

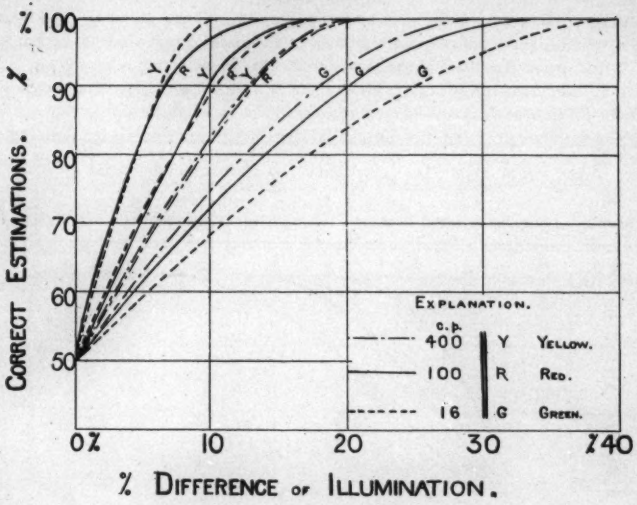
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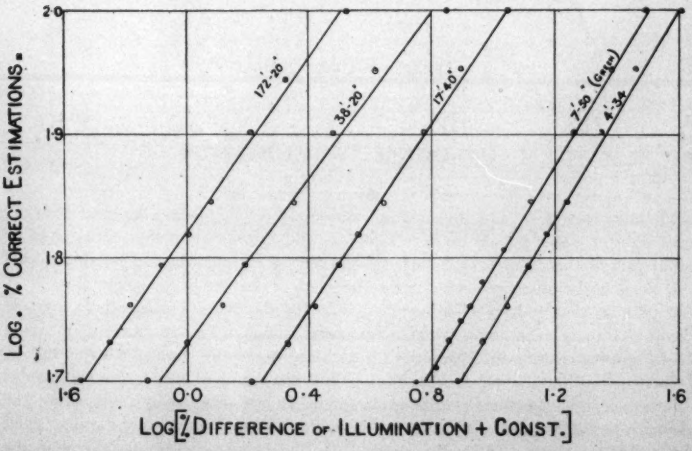
Fig. 10.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1·6
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1·6
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172·20'
36·20'
17·40'
7·30" (GREY)
4·34

Fig. 11.

case of the latter. It is difficult to offer any explanation of what may possibly be an inconsistency due to some unforeseen variation of the conditions of observation.

Owing to the limits of this lecture, it is not possible to consider the curves obtained for other areas of the retina. It must suffice to mention that, while all the curves belong to the same family, there are considerable differences in one of the constants over larger areas in the case of coloured lights of various intensities. Now, if any of the preceding curves of Figs. 6, 7, 9 and 10 are plotted logarithmically with respect to the percentage of correct estimations E as ordinates and the percentage difference of brightness B plus a constant as abscissae, they can be reduced to straight lines all of nearly the same slope as indicated in Fig. 11. The equation of the original curves is accordingly of the form $E = a(B + b)^n$ where a and b are constants and n represents the slope of the logarithmic curve. The values of n vary from ·32 to about ·46, the average being 0·4. Thus,

$$E = a(B + b)^{0\cdot4} \qquad \ldots\ldots(1).$$

But both a and b appear to be functions of the angular diameter of the retinal area illuminated. Thus in Fig. 12 where the values of log a are plotted as abscissae

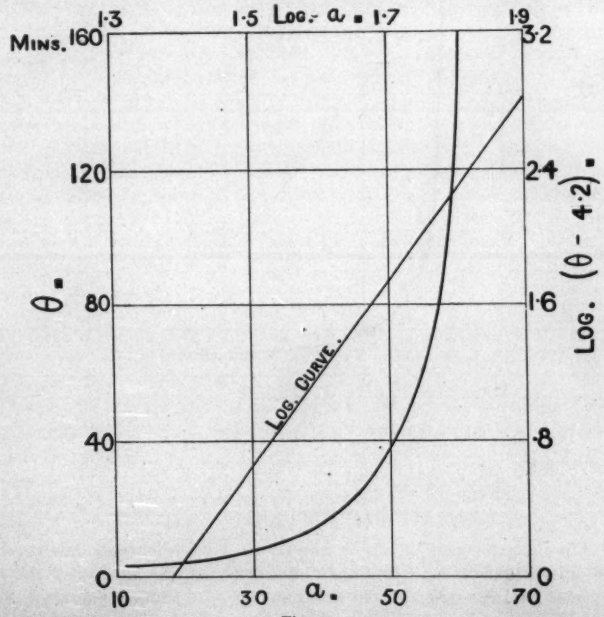

Fig. 12.

and $\log(\theta - 4{\cdot}2)$ as ordinates, the curve is a straight line and for the equation of the original curve there is obtained the expression

$$a = 24(\theta - 4{\cdot}2)^{0{\cdot}18} \qquad \ldots.(2).$$

Similarly the variation of $\log b$ with respect to $\log(\theta - 4{\cdot}2)$ is indicated in Fig. 13 and from this there is obtained a similar equation

$$b = 3{\cdot}31(\theta - 4{\cdot}2)^{-0{\cdot}3} \qquad \ldots.(3).$$

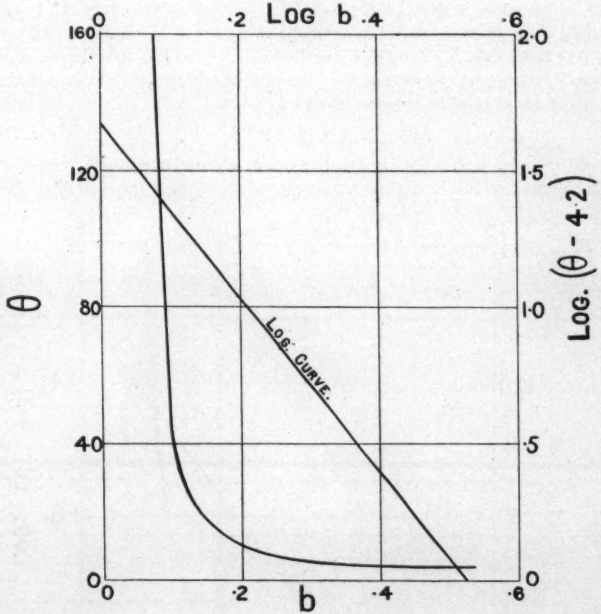

Fig. 13.

Substituting the values of a and b, equations (2) and (3), in equation (1), there is obtained the empirical expression

$$E = 24(\theta - 4{\cdot}2)^{0{\cdot}18}[B + 3{\cdot}31(\theta - 4{\cdot}2)^{-0{\cdot}3}]^{0{\cdot}4} \qquad \ldots.(4),$$

where E is the percentage of correct estimation of the brightness difference, B is the percentage difference of brightness, and θ is the angular area of the retina illuminated, for areas ranging from about 40° down to about 5 minutes.

From equation (4) it will be seen that theoretically absolute uncertainty in the

appreciation of difference in brightness is reached when $\theta = 4{\cdot}2$ minutes. This, however, can only be true within limits, since the form of the image, that is, the contrast of the image with the black background, is always distinguishable.

Difficulty was experienced in dealing with the very small holes, especially with coloured lights, such as blue, when the illumination was low, owing to the difficulty of maintaining direct vision and to a want of control of accommodation. The smallest hole that was attempted was $4'\ 30''$. From the mean of three series of observations on such a hole the value of B obtained was about 34 per cent. and the curve was asymptotic for the value of about 4 minutes, as indicated by equation (4).

If two circular images are made to overlap, say by means of a Wollaston double image prism, the overlapping portion will appear distinctly brighter than the remaining portions, provided it covers an angular area of the retina considerably greater than four minutes. If, however, two small areas are made to overlap by an amount subtending not more than four minutes, it will be difficult to detect in the whole combined image any variation of the illumination, although the contour or form of the combined image is distinctly defined. This seems to imply that the four-minute foveola is sensitive to form, although comparatively insensitive within certain limits to variations of intensity. In the case of two overlapping circular areas which subtended an angle of seven minutes, an increase in the intensity of the overlapping portion was just definitely appreciable.

When the ends of two images of a long narrow slot, whose horizontal width subtended less than four minutes, were brought into contact with one another and then made to overlap by varying amounts, a difference in the brightness of the overlapping portion was perceptible when the overlap was somewhat less than 4 minutes. These experiments were made with slits of various widths and the results are indicated in Fig. 14, the ordinates of which represent the angular width of the slit and the abscissae the angular amount of the overlap at which increased brightness became just apparent. For the larger widths of holes down to about $4\frac{1}{2}$ minutes the overlap was constant, being about $1{\cdot}6$ minutes. Since these slits were wider than the 4-minute foveola, it is to be expected that the change of brightness would be visible on either side of the 4-minute area. As the width is reduced, the overlap increases, being between 3 and 4 minutes when the width is under 2 minutes.

Whatever the reason may be, it would appear that in direct vision large variations in the brightness of images covering these small retinal areas cannot be appreciated, although the form of the image may be clearly distinguished when the intensity is sufficiently great. An investigation of the histological details of this particular area is desirable. The preparation of micro-sections of the fovea centralis was commenced for the purpose of this paper, but the work was unfortunately interrupted.

Before discussing the resolving power of the eye, it is desirable to understand clearly what is meant by precision or accuracy, and by adjustment or setting. Suppose that in bringing two circles into contact the ten consecutive readings

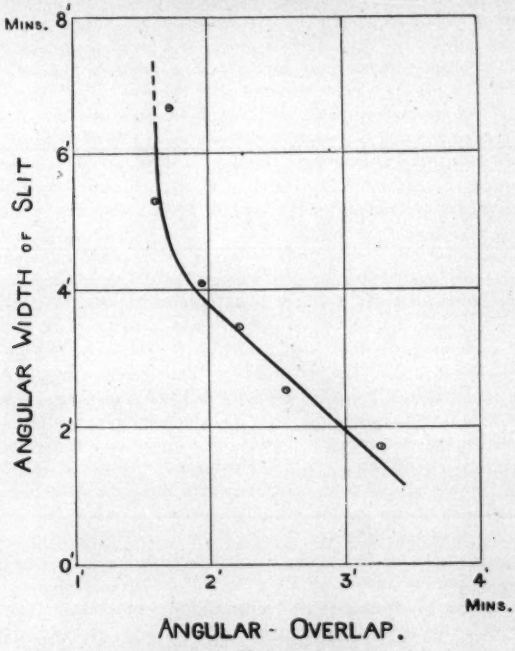
MINS. 8'
6'
4'
2'
0'
ANGULAR WIDTH OF SLIT
1'
2'
3'
4'
MINS.
ANGULAR - OVERLAP.

Fig. 14.

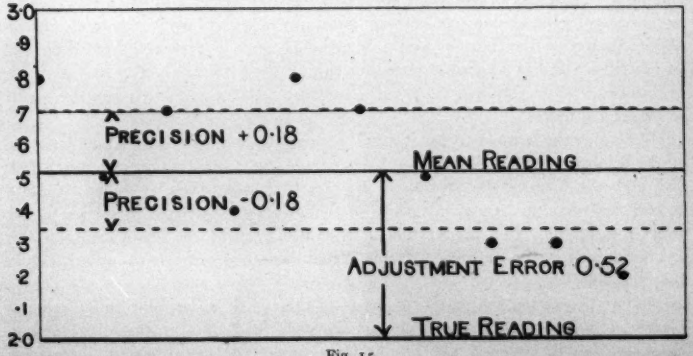
30
9
8
7
6
5
4
3
2
1
20
PRECISION +0·18
MEAN READING
PRECISION -0·18
ADJUSTMENT ERROR 0·52
TRUE READING

Fig. 15.

of column *A* are obtained. The arithmetical mean of these is $B = 2{\cdot}52$. Column *C* indicates the differences from the mean of the individual readings and *D* is the mean of these differences. These data are plotted in the diagram, Fig. 15. Suppose

A	*C*
Reading	Difference from mean
2·8	0·28
2·5	0·02
2·7	0·18
2·4	0·12
2·8	0·28
2·7	0·18
2·5	0·02
2·3	0·22
2·3	0·22
2·2	0·32
(*B*) Mean 2·52	(*D*) Mean 0·18

the true reading for the contact of the circles is 2·0; then the adjustment error is $2{\cdot}52 - 2 = 0{\cdot}52$, and the precision *D* is $\pm$ 0·18. That is, although the adjustment may be wrong by 0·52 units, the observer in the series of ten readings repeats the same setting with an average error of $\pm$ 0·18 units.

Now, although the precision can be improved only by increased care in observation or improvement of the conditions, an allowance for the adjustment error may sometimes be made. For example, many observers suffer from premature judgment and, if, in a particular type of observation, the amount of the error is known, the observer may make some allowance at each observation for his known bias. Unless the operator is particularly skilful, it is unwise to attempt any compensation of bias. Throughout the tests to be described, compensation of the bias was carefully avoided and in any case was difficult, as the observer was unaware of the scale readings, which were noted by a separate observer in all cases.

In tests of the accuracy with which illuminated slots may be brought into contact or made to overlap, the object to be observed has generally the form of a grating. It may consist of slits cut on a silvered surface or ruled lines or fine wires. Such arrangements have the disadvantage that the spacing required is very small unless the eye is placed at a great distance, and to vary the angular spacing it is necessary to vary the distance, unless some minifying optical system is introduced. At a distance of 100 inches a linear spacing of 0·03 inch is required to give an angular spacing of one minute.

In the following experiments two holes or slits in a diaphragm were viewed at a distance of 100 inches through a Wollaston prism which produced double images of each hole. Of the four holes thus seen, the two middle ones could be brought into contact, made to overlap, or moved out of contact on the other side by a suitable longitudinal displacement of the Wollaston prism along a scale. As the angle of duplication of the Wollaston double image prism was 2176 seconds, holes of a very considerable size could be employed. In some of the experiments the holes were illuminated by direct light diffused by a ground glass screen and in others by diffuse light reflected from white surfaces.

When conducting an experiment of this kind, using circular holes, the appearance may be divided into three definite stages.

In the first stage the two holes just appear to touch each other. Suppose the true angular diameter of the hole is θ and the apparent diameter, as measured by the Wollaston prism, is θ_1. Then, if the retinal image is perfect and the contact is exactly correct, the angular separation of the two images should be equal to the true angular diameter θ. Owing to diffraction and aberrational effects, it might be expected that the retinal image would be larger than the true image by an amount $d\theta$. Actually θ_1 may in some cases be several times greater than θ, and under certain circumstances slightly less. But the extent to which the eye can appreciate the effects of diffraction and chromatism is very uncertain. It is difficult to reconcile calculated and actual values. When the hole is very small the observation becomes increasingly uncertain, owing to the difficulty of controlling the direct vision and owing to diminished control of accommodation. With regard to the latter, it is possible that, in the case of very small and comparatively low intensity simple images, the eye automatically strives to spread the image over a greater area of the foveola. On the other hand, it may be supposed that the stimulation of the retina is insufficient to produce the reflex accommodation control of the crystalline lens.

In the second stage the appearance of two intersecting circles is lost and the image assumes an oval form, no discontinuity in the outline being observable.

In the third stage the two images overlap more or less accurately and the form is indistinguishable from that of a single circular spot.

Increased accuracy in overlap observations may be obtained by rapidly interrupting the images alternately, thus producing a winking appearance at the edges when the superposition of the images is not perfect. Artificial aids of this kind, however, were not used in the experiments. It may be of interest to observe that winking devices were made considerable use of in certain German army instruments.

Some of the results of observations on circular holes of various diameters are indicated in Fig. 16, in which ordinates represent the true angular diameters θ expressed in minutes of angle, and the abscissae the observed angular values θ_1, as measured by the Wollaston prism. Curve A refers to the contact observations which were made both for images approaching and receding from each other. The differences between these two sets of observations were negligibly small. Theoretically the results should lie upon the diagonal line B which passes through zero. If the measured angle θ_1 is larger than the actual angle θ, the curve A will lie below the line B. It will be seen that the difference is very constant over the range of holes down to an angular diameter of 3 minutes. If the difference is due to effects such as diffraction, irradiation, and dioptric aberrations, it is to be expected that the image diameter would be greater than the object diameter, but in other curves taken under slightly different conditions, the reverse is actually the case. Defective judgment, on the other hand, might act in either direction but the tendency is generally to make θ_1 greater than θ.

As the object diameter approaches 3 minutes, the curve commences to deviate from the theoretical value and the image diameter is no longer directly proportional to the object diameter but tends to become constant. For holes below one minute the centres of the two images subtend a constant amount of two minutes. That is, the image diameter, as appreciated by the small portion of the retina in question, is actually twice the diameter of the object, whereas in the case of the images covering a larger area of the retina, the difference is a little more than 20 seconds.

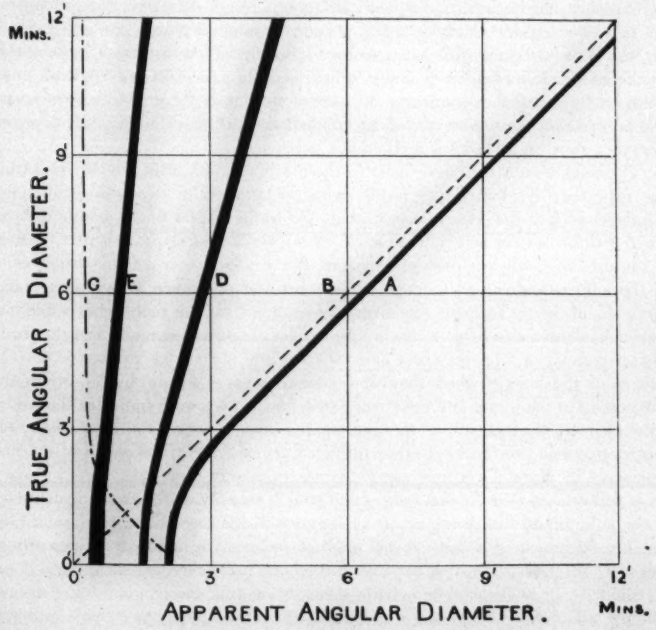

Fig. 16.

This does not imply that the small area is less capable of judging contacts than the large area: it may be that the small area is equally accurate but that it is the images that are larger. This curve would appear to suggest that diffraction and dioptric aberrations are not the determining factors of first importance. The differences in question do not correspond with those calculated and a considerable variation of the pupil diameter does not very appreciably affect the general results.

In the case of the very small images it is possible that the minimum size is determined by the grain or texture of the retina, as suggested by Descartes and

many later investigators. But if this is the case it might be expected that the constant amount of about two minutes in the particular experiments described would be applicable to larger retinal areas. This, however, is not the case, because from the 3 minute hole to the 11 minute hole the difference between θ and θ_1 is only 20 seconds in this particular series; that is, only one third of the angle subtended by the retinal cones. It is possible that, on the foveola or the 4 minute area of the fovea centralis on the line of direct vision, even if the cone elements were smaller than elsewhere, the light insulation might be defective, thus permitting the rays entering one element to diffuse into the adjoining ones.

It is interesting to note that it seems to be impossible to form a point image on the retina. The minimum image subtends quite a considerable angle of about 2 minutes, the calculated amount on a diffraction basis for an ideal point image and a minimum pupil diameter of 2 mm., being 1 minute of angle, and very considerably more on a theoretical chromatic basis.

The thickness of the curve denotes the mean accuracy with which the setting was repeated. Evidently this factor, which is practically constant throughout, is independent of the error of adjustment. From the largest to the 3 minute hole, the repetition accuracy is a little more than half the adjustment error, whereas in the case of holes less than one minute, the accuracy is less than 10 per cent. of the adjustment error. Curve *C* represents the difference between the ideal curve *A* and the actual *B*. It represents the apparent angular increase in the retinal image diameter relatively to that of the object, assuming that the images are in perfect contact on the retina. Curve *D* indicates the angles subtended by the centres of the images when they first assume an oval form; that is, when the indentation of the upper and lower edges disappears. Curve *E* indicates the angles subtended by the centres of the overlapping images when the oval shape has disappeared and the form appears truly circular. As before, the width of the line represents the accuracy within which the readings were repeated.

It will be seen that the accuracy of adjustment steadily increases as the diameter of the hole diminishes, being about $1\frac{1}{2}$ minutes in the case of the 10 minute hole and 0·5 minute in the case of the smallest holes. The accuracy of repetition, however, decreases slightly as the size of the hole diminishes.

Similar curves were obtained for the contact and overlap of long slits but although the overlap results were practically the same, the contact results differed appreciably. The curve only commenced to change at the $2\frac{1}{2}$ minute slit. For small slits the constant apparent width was 2 minutes, as in the case of the circular images, but for the large slits from 11 minutes to $2\frac{1}{2}$ minutes, the constant adjustment error was about 40 seconds, as compared with 20 seconds for the circular images. Possibly the form of circles in contact assists the judgment.

In these experiments the illumination was constant and such that practically no star appearance of the image was observable. In front of the eye there was placed a circular diaphragm of 2 mm. diameter. In the following experiments no diaphragm was placed in front of the eye and each hole was tested when illuminated by light of various intensities. The intensity was varied by means of absorbing

screens, the values of which were known. In the case of the strongest intensities the star appearance was very marked when the diameter of the hole was small and the difficulty of observation was considerable. This effect was not so marked in the case of the larger holes.

Fig. 17 shows one series of results obtained for a hole of 67 seconds angular diameter. Ordinates represent the number of absorbing screens between the object and the illuminating source and abscissae the apparent angular separation of the images in contact, as measured by the Wollaston prism. At the point marked *S*, which is the place where the curve rapidly tends to become vertical, the illumination was such that the stellate appearance had just disappeared. At the lower end *R* of the full line portion of the curve, the illumination was so small that difficulty commenced to be experienced in focussing the image definitely, the point being just above what is called the threshold of vision, which is represented

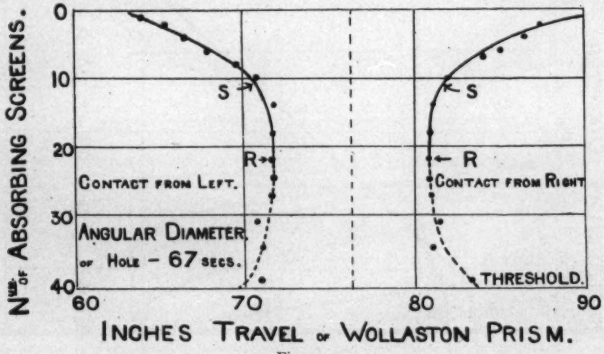

Fig. 17.

by the dotted portion of the curve. It will be observed that the curve is fairly definite at the threshold and that the observed angle increases in value.

Similar results for holes of various diameters are combined in Fig. 18, the ordinates of which represent the true angular diameters of the holes and the abscissae the apparent angular separations of the images in contact. In the green curve *A*, the illumination was a minimum, the data being obtained from the lowest parts *R* of the full line curve in the previous Fig. 17. In the red curve *B* the illumination was such that the star appearance just disappeared, the data being obtained from the parts *S* of the curve, Fig. 17. In the black curve *C*, the illumination was a maximum and the star appearance was marked. For the larger holes the curve as before is parallel to the ideal line which passes through zero, but in this case in which the full aperture of the pupil was used, the curve lies above the line, whereas in the previous similar diagram when a 2 mm. diaphragm was used the curve lay below the line. When the diameter of the hole subtends 5 to

6 minutes, the high illumination curve *C* deviates rapidly but it should be remembered that the stellate form of the image makes the observations somewhat indefinite. Later it approximates again in the case of the very small holes to the minimum figure of the curves *A* and *B*. These two curves deviate when the hole is about 4 minutes and for smaller holes the minimum apparent diameter is 2 minutes for the medium illumination curve *B* and 1·7 minutes for the low illumination curve *A*.

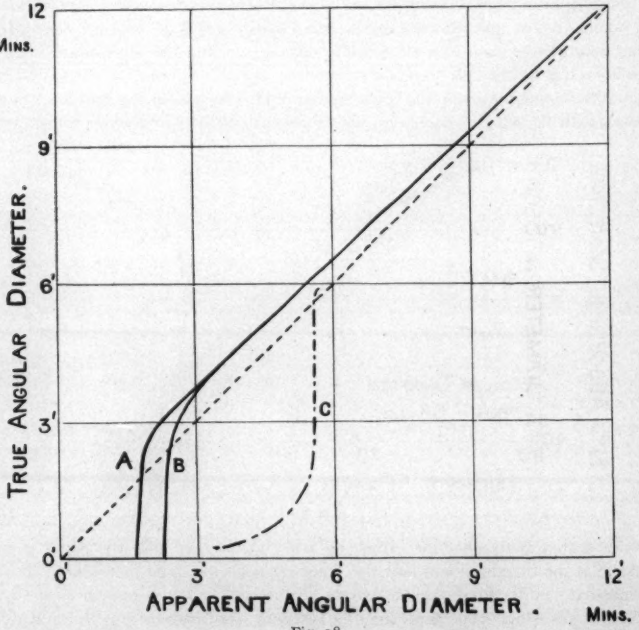

Fig. 18.

A clearer idea of the variation of the retinal image diameter with the intensity of the illumination in the case of objects of various sizes is indicated by Fig. 19, in which the ordinates represent the apparent angular diameters of the retinal images and the abscissae the intensity of illumination. In the case of the smallest diameter hole, which subtended an angle of 12·3 seconds, it will be seen that the size of the retinal image increased almost directly with the intensity. A possible explanation is that the diffraction and aberrational zones become increasingly evident as the illumination is increased, but the form of the curve hardly suggests this, especially if it is a question of contrast. There is the alternative explanation

that the light insulation of the retinal elements decreases towards the centre of the fovea centralis, with a consequent spreading of the image; that is, the irradiation in the strict sense of the word may increase.

As the size of the image is increased to over 3 minutes, the form of the curve changes; a small increase of the intensity produces at first a comparatively large increase in the image size, but the curve tends rapidly to become horizontal; that is to say, there is then no increase of the image size as the intensity is increased.

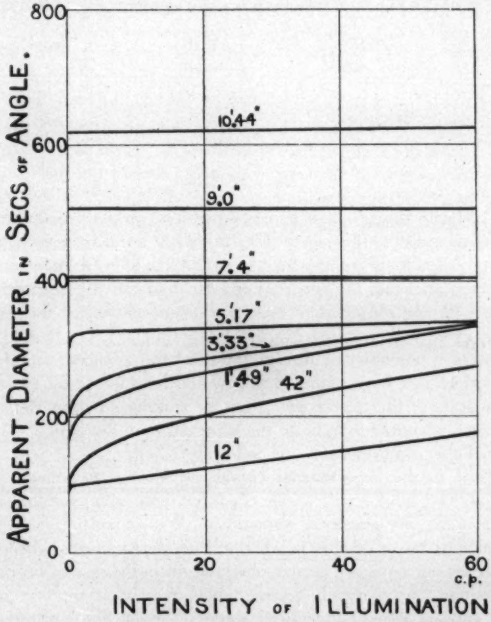

Fig 19.

When the diameter of the object was 5'.17" the angular diameter of the retinal image near the threshold of vision was the same as that of the object and the diameter did not vary with the illumination, since the curve is practically a horizontal line. For larger holes the curves are also horizontal lines. Thus, between the curves for the $3\frac{1}{2}$ minute and the 5 minute holes, there is evidently a distinct change in the relationship of the apparent image diameter to the illumination, thus suggesting, as in the previous experiments, a gradual change in the functions of the fovea centralis at an angular diameter of about 4 minutes.

If the variations in the case of the small objects are attributable to diffraction

and aberrational effects, it is not easy to explain why there should be no such variations in the case of objects subtending more than 4 to 5 minutes. With regard to the alternative explanation, it would appear that the retinal elements are more closely crowded together towards the centre of the fovea centralis, where they are stated to be in actual contact, whereas in the more peripheral parts of the macula lutea, the elements are separated by the extensions of the pigmentum epithelium, there being indeed sufficient space at least in the eyes of the lower animals for the movement of the pigment cells between the elements over their whole length.

It was hoped to include in this lecture the results of experiments on the accuracy of coincidence observations, but owing to the extent of the material available it is necessary to reserve it for a third lecture, in which the general theory of vision will also be discussed.

DISCUSSION

Mr L. C. Martin thought that it was difficult to pass a final judgment on the conclusions of the paper as the experimental work had not been very fully described. There was one point, however, to which he would like to draw attention. Referring to Fig. 19 it appeared that the apparent angular diameter of the retinal image did not decrease with the illumination near zero illumination for the larger areas. This did not agree with his (Mr Martin's) experiments*, for he had found that when the intensity approached threshold values the apparent angular diameter decreased until quite a large patch of light would tend to appear almost star like. Probably the scale of the curves would not be suitable to show the effect but it might have been advisable to indicate the uncertainty in the figure.

Mr Martin also communicated the following.

With regard to the experimental curves expressing percentage numbers of correct estimations I think it will be conceded that these only meet the 100 per cent. line *by reason of the finite number of observations*. We are not told if they are due to one or several workers, or, if several, whether they were educated observers or not. It is clear that the percentage of correct observations in the case of a careless subject would be less, and thus give an increasing tendency to an asymptotic nature of the curve. This appears a small and rather academic point, but it seems to point to the alliance of the observations and the "probability" curve.

My own work did not involve experiments on areas so small as 4 minutes but I have drawn attention to the work of Druault with a Snellen test chart where observations are necessarily made with dark letters on a light ground. Some change of slope in the logarithmic curve seems to be shown at about 4 minutes subtense but it is not clear whether the work is very trustworthy or not. It would be well worth while to verify these results.

Mr T. Smith said that he had been most interested in reading Mr French's

* "A Study on the Performance of Night Glasses," *Bulletin*, No. 3, Dept. of Scientific and Industrial Research.

paper. He particularly welcomed the experimental results which had been given, and which evidently represented a volume of work that would perhaps only be recognised by those who had some experience in attacking such problems. It was however necessary to state the actual brightness of illumination of the surfaces examined to render the results of value to other workers. The candle power of the lamp employed was of no importance save in so far as it might indicate the probable spectral distribution of energy it gave out. The important quantity is the energy falling in a given time on unit area of the surface.

The results of the observations were of great interest, but the way in which they had been analysed was open to a good deal of criticism. The curves of Fig. 7 were smoothed curves of the same general character as those of Fig. 6. The halves included in the figure however had been drawn in such a way that if the two parts were shown they would meet at a sharp angle, a state of affairs which certainly does not correspond with the facts. It would imply for example a much greater accuracy in photometry than is attained. The lower parts of the curves require rounding so as to touch the line $E = 50$. By losing sight of the symmetry which is an essential feature of a correct curve the author had been led to express his results in a form which was meaningless from the point of view of physical interpretation. Fig. A shows by the continuous black lines the curves plotted from the author's formula (4) for the cases illustrated in Fig. 7, from which the dotted curves have been traced

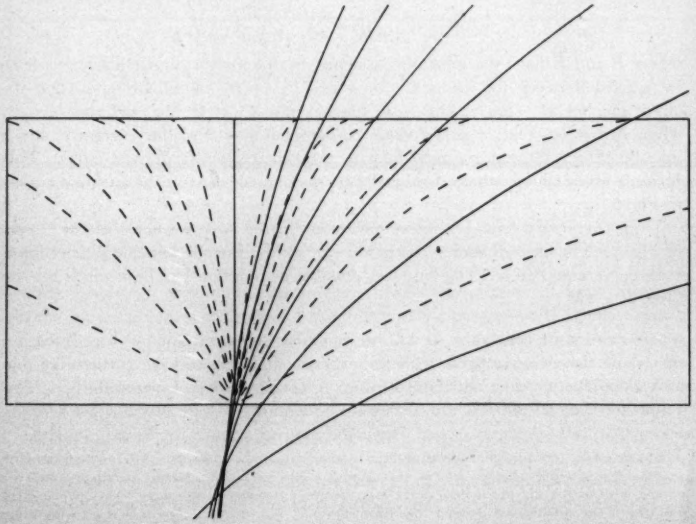

Fig. A.

for both left and right-hand sides. There are two faults besides lack of symmetry in the curves plotted from the formula which are at once obvious—they cut the line $E = 100$ at a large angle instead of being tangential, and they fall below $E = 50$—indeed as θ approaches 4·2 closely they tend to coincide with $E = 0$—a result which can only be construed to mean that the formula does not fit the facts correctly. However it will be seen from the figure that if only one half of the experimental curve is considered there is fair agreement for most of the curves shown, though the errors for the two smallest apertures probably exceed the experimental errors. If this formula were adopted it would be necessary to consider, in addition to the critical angle of about 4·2 minutes, another critical angle of about 3 degrees beyond which the formula did not apply. For if this formula could be applied further the percentage difference of illumination at which absolute certainty would be reached would further decrease as follows:

	$\theta = 5°$	10°	20°	40°
Percentage difference of illumination ...	2·74	1·51	1·07	0·75

There is no reason to believe that such figures could be justified.

It is desirable to see whether these experimental results justify the introduction of a number of critical angles which mark points of discontinuity where one law gives place to another. This may be done by attempting in the first place to fit a formula to the curves of Figs. 6 and 7 free from the features to which exception has been taken. A simple formula which may be considered in connexion with Fig. 6 is*

$$E = 100 - 50 \cdot \frac{b^2(a^2 - B^2)^2}{a^4(b^2 - B^2) + 2(a^2 - b^2)^2 B^2}$$

where E and B have the meanings assigned by the author, and the formula is to be applied between the limits $B = -a$ and $B = +a$. It will be noted that the curve touches $E = 100$ at $B = \pm a$, touches $E = 50$ at $B = 0$, and that $E = 75$ when $B = \pm b$. Thus a is the value corresponding to absolute certainty, and b may be chosen to secure a good fit with the experimentally derived curve. This formula gives curves which can hardly be distinguished from the author's curves in Fig. 6.

Fig. B shows a family of these curves compared with a tracing of the curves of Fig. 7. They are all similar, with $b = \cdot 3a$, so that the one variable a determines each curve completely. The simplest possible assumption has been made for the form of a, viz.:

$$a = c + d/\theta$$

where c and d are constants. It will be seen from an examination of Fig. B that on the whole these curves agree with the experimental curves (even considering one half alone) better than do those of Fig. A constructed from formula (4). The values for the constants as the curves are drawn are $c = 2{\cdot}86$ and $d = 2$ when θ is

* Commander Baker has suggested that the reason for providing for a second constant b should be given. Mr French's curves of Figs. 6 and 7 appeared to be decidedly more pointed or concave than the probability curve or most simple curves with one arbitrary parameter, and the power of adjusting for the middle point was therefore considered necessary. This is evidently essential if the differences between the three curves of Fig. 6 which have practically the same terminal points are real and not the results of experimental errors.

expressed in degrees, or $d = 120$ if θ is in minutes. If this formula is adopted as a reasonable representation of the facts both the critical angles disappear, for however large the angle may be the percentage difference of illumination for which absolute certainty of discernment exists cannot exceed 2·86, which is in satisfactory agreement with the results reached in the experiments with the range-finder. These limiting points are marked on Fig. B by arrows. If any lower discontinuity is introduced it must be based on the impossibility of discerning the two halves of the illuminated circle as separate sources of light when θ falls below some definite value. This cannot be 4·2 minutes as the dividing line was clearly seen for this aperture. The introduction of such a discontinuity appears quite unnecessary in view of the rapid fall of the curve given by this formula when θ is very small. But further, this alternative treatment has led to a law easily interpretable physically, a result which could hardly be derived from the author's equation. It may be noted that the amount of light received is proportional to $c\theta^2 + d\theta$, which is very suggestive

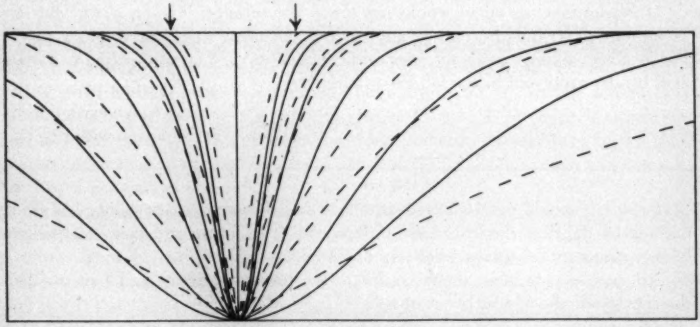

Fig. B.

in connexion with the views put forward before this Society recently by Chalmers, since the term in θ is of paramount importance when θ is small and that in θ^2 when θ is larger. In any case the experimental results here considered give no support to the view that there is any discontinuity in the appreciation of light intensities as the source becomes too large for the image to fall entirely on the yellow spot of the eye.

Instructor-Commander T. Y. Baker said he could not agree with the author on one point, namely, that in making contact between two bright circles the same accuracy of setting would be obtained whether the initial position was one of overlapping or of separation. In 1912 he himself had experimented with two circles illuminated from behind; the circles were distinctly larger than those used by Mr French, the diameters subtended at the eye varying from about 40 minutes to 3 or 4 degrees. When the circles were initially overlapping and moved by the observer until the overlap apparently just disappeared, an accuracy of about 10 seconds

was obtained with the small circles and about 5 seconds with the large ones. When, however, contact was made from an originally separated position, the accuracy was about 35 or 40 seconds.

Commander Baker also communicated the following:

While agreeing with Mr Smith that the formula (4), put forward by Mr French for the percentage of correct estimation of the brightness difference B, when θ represents the angular area of the retina illuminated, is inadequate, yet I think that Mr Smith's counter proposal is not necessarily the only one that can be used. The curve shown in Fig. 6 bears a very strong resemblance to the probability curve (see Airy, *Theory of Errors of Observation*, p. 17). The equation of this curve transformed so as to fit in with Mr French's arguments can be written

$$100 - E = 50e^{-B^2/c^2}$$

where E and B have the same meaning as before; e is a parameter dependent upon the area of the bright patch on the retina.

It is true that the curve would not touch the certainty line ($E = 100$) but be asymptotic to it; but the rate of approach to the asymptote is exceedingly rapid. For instance, taking c equal to 1 per cent., the values of E corresponding to values of B can be expressed in the following table:

B	E	
0 %	50	% absolute uncertainty.
1 ,,	71·6	,,
2 ,,	99·1	,,
3 ,,	99·994	,,
4 ,,	99·999995	,,

I think it would not be unreasonable to say that absolute certainty has been reached by the time the brightness difference is 3 per cent. and that $c = 1$ might be the parameter for the curve shown in Fig. 6.

I do not care to insist on this value at the present moment as I have neither the curves nor the figures before me.

The values of c would best be estimated from a measured intermediate value of E, say 75 per cent.

Probably if this curve were examined it could be made to fit the series of experimental results shown in Mr French's figures and it would possess the advantage of conforming to the generally accepted theory of errors of observation with which the whole subject is closely allied.

Mr W. Swaine, referring to Commander Baker's observation on the precision of contact setting of circles, remarked that, although not mentioned in the present paper, results were obtained which verified the results Commander Baker had communicated at the Optical Convention of 1912. From Mr French's results there was appreciable overlap of the two circles before increased brightness was observed, and therefore the point of appearance or disappearance of a bright overlap in the experiments in question could not be taken as a criterion of contact. For the smallest holes the two circles fused into an oval shape, no brighter overlap ever being observable. He mentioned this fact because in many of the experiments described he had been working under Mr French.

Mr Smith's equation obviously expressed more accurately the empirical relation between the various quantities. The curves were drawn from a large number of very careful observations, but, as Mr Smith remarked, the number was not adequate to determine with sufficient accuracy the symmetrical rounding off at the 50 per cent. mark.

With regard to Mr Martin's remarks it should be observed that the low illumination of the holes was not at the absolute threshold, but at a very appreciable distance from it. This might eliminate the apparent discrepancy between the results obtained by the author and by Mr Martin.

Mr J. Guild communicated the following remarks:

Mr French's paper is of great interest, and the experimental results, embodying as they evidently do a vast amount of careful and tedious observations, are of great value.

On page 10 of the paper the author suggests that the diminishing sensitiveness to changes of brightness with diminishing area may bear a definite relation to the ratio of the rods and cones. In this connexion it is interesting to recall that Prof. Abney and the late Prof. Watson established the existence of two types of eye; in one, the prevalent type, the rods are entirely absent in the fovea; but in the other type the fovea either contains an appreciable number of rods, or else certain processes partaking of the properties of both rods and cones. If Mr French's measurements were repeated by an observer known to have the second type of retina, a marked difference in the curves should result if the ratio of the rods and cones is the important factor.

With regard to the experiments with different coloured lights there is a possible explanation of the anomalous variations of the sensitivity with brightness. The Fechner fraction, though approximately constant for medium intensities, has a minimum value at an illumination of 6 or 7 metre candles. At illuminations above and below this the sensitivity falls off. If, therefore, the brightnesses with the red filter were such that the two highest of them were well above the point of maximum sensitivity, and if, in the case of the green filter, the two lowest illuminations were well below the point of maximum sensitivity, the observed effects could be accounted for. It is impossible, in the absence of information as to the actual brightnesses during the experiments, to say whether such conditions held or not; but with certain filters, the red and yellow may easily have been much brighter than the green. Moreover, although I am not aware of any information on the point, the maximum sensitivity may occur at very different brightnesses for different colours.

The experiments on the contact of circles and slits are of great interest. It appears to me that the explanation of the measured diameter being too large with the full pupil and too small with the restricted pupil* in the case of the larger diameters is simply that two sources of error are here balanced against each

* By mistake in writing out the communication the phenomena have been put in the wrong order. The measured diameter is too small with full aperture, and the explanation of the next few sentences accounts for this fact.

other. I think the observer's tendency in such a setting is always to make the discs overlap, i.e. to get too small a value for the diameter. On the other hand diffraction effects tend to make the images too large. With the small aperture this second effect more than balances the overlap effect and so the result is a little too large; but with the full aperture the second effect is much less and the tendency to overlap predominates, and the result is too small.

The behaviour with small discs is most interesting and the reason for the minimum measured diameter of 2 minutes, however small the actual diameter may be, is not immediately obvious. I think however that the tentative explanation given at the top of page 20 breaks down on examination. It would involve for instance that a black spot on a white ground would be invisible if its angular diameter were less than 2 minutes. This is not so. Further the same results are obtained when the holes are replaced by slits (see page 20, para. 5) and here of course only a small portion of the slit can lie on the foveal region in which the author supposes the light insulation to be defective.

A similar series of experiments with black spots on a white background would obviously throw a great deal of light on the question. Miss A. B. Dale and I had hoped to perform such experiments before the meeting but were unfortunately unable to do so in time.

The author communicated the following reply:

I appreciate the difficulty the members must have experienced in discussing these lectures owing to the lack of the actual data from which my diagrams have been made. To have included the data in which thousands of observations are involved was impracticable even in the limits of three papers. If any member is particularly interested in any special section I would endeavour to arrange the corresponding data and place them at his service.

In reply to Mr Martin, the lowest illuminations referred to in the paper, as explained by Mr Swaine, were above the threshold values.

From Mr Martin's reference to the diminution of size of an image as the threshold is approached and to its final star appearance, I presume the whole pupil was being used and not the central core. In certain threshold value experiments not referred to in these lectures, I have observed that, just at the threshold of vision, a small image expands to an ill-defined violet patch. This appears to be the reverse of Mr Martin's observation. It is possible that owing to the use of only the core of the crystalline lens, my threshold values were actually lower than Mr Martin's. The increase in size observed by me may be due to the quantity of light being insufficient to stimulate the reflex controls of the crystalline lens, with the result that the focus of the retinal image is defective. Lord Rayleigh has, I think, already made a somewhat similar observation.

Four observers took part in the Photometric Certainty of Observations experiments—three men including myself, and one lady. The majority of the observations were made by Miss Robertson, who is a particularly consistent observer. Sets of check readings only were taken by me.

When replying to Mr Smith, I shall deal with Mr Martin's other point regarding the extreme ends of the curve.

With regard to Mr Martin's concluding remarks, I have made a considerable number of observations upon the limits of appreciation of form. These are not referred to in the lecture. When using dark figures on a white background, seven geometrical shapes were tested. Although the absolute certainty angle depended somewhat upon the shape, the general conclusion arrived at was, that 100 per cent. certainty was attained when the angle subtended by the object was about 4 minutes. This appears to confirm Druault's work, of which I was unaware.

Now in reply to Mr Smith. I think our difference of opinion is only one of degree and capable of adjustment.

First let us consider the absolute certainty end of the curve, Fig. 6. If the difference between the two photometric fields is great, a careful observer could not fail to discriminate between them. In an extreme case it is hardly a question of a finite number of observations, as Mr Martin suggested.

At the absolute uncertainty end however the errors are great. To place any reliance on the observations at this end and to some extent on the other end of the curve is undesirable.

So far from suggesting an accuracy that is unattainable in photometric observations, my endeavour has been to exclude the ends of the curves and to derive only very general results from the intermediate portions of the curves. Further, I have stated definitely that the equation applies only to the intermediate portion, and it will be observed also that I purposely excluded the very small angular values on account of the necessarily large errors of observation in these regions.

It would have been easy to extend my formulae to include more extreme portions of the curves, but in any case, I regard the measured data as more important than a formula whose application can only be limited.

That the curve becomes rapidly tangential to the absolute uncertainty line is doubtless a safe deduction, but the data do not show it, and in any case, the change of curvature is probably so great that it could not be clearly represented on the scale of the diagram.

Briefly, I think Mr Smith has regarded the extreme portions of the curves as of importance, whereas I have not, as the data do not permit it.

With regard to Mr Smith's very interesting equation, my principal objection to it is that it does not appear to fit the observed data at all well. As Mr Smith states his formula gives curves which can hardly be distinguished from mine, I fear there must be some misunderstanding. I can hardly agree with Mr Smith's plotting of my curves. He shows a divergence which I think is much too great.

It seems to me that his equation has been so built up as to give the desired terminal values and form, but it does not fit well in other respects. Mr Smith appears to have built up a formula to which the data must be fitted, whereas I have tried to deduce an approximate formula from the measured data; but again I would emphasise that it is the data that are of importance. The equation has no very precise meaning, because the curves cannot apparently be expressed by one general equation.

In reply to Commander Baker, the results obtained from small apertures appear to be different from those obtained from the larger apertures. In experiments not included in this lecture, with larger apertures comparable with those of Commander Baker, the precision of approaching contact was less than that of withdrawing, the respective figures being 4·7 and 3·4 seconds in one typical set of experiments. This to some extent confirms Commander Baker's results. The smaller difference obtained by me may probably be attributed to the elimination of stellate appearance by the use of a pupil diaphragm. Unless the stellate appearance is eliminated it is difficult to make precise observations.

I can also confirm that greater precision of setting is obtainable in the case of larger holes when of the order of those used by Commander Baker. It should be remembered that only holes small in comparison with these were included in the lecture.

In the case of large holes an increase in the illumination of the overlapping portions is observable at the upper and lower extremities of the overlap. This serves as a guide in setting. In the case of small holes there is no such guide for the reason stated in the lecture, and a difference of precision between approaching and withdrawing contacts is not marked.

With regard to Mr Guild's remark that for large holes the measured diameter is too large with the full pupil, I fear there must be some mistake*. In the lecture it is stated that "if the measured angle θ_1 is larger than the actual angle θ, the curve A will lie below the line B" (that is the diagonal through zero), and again later, "For the larger holes the curve as before is parallel to the ideal line which passes through zero, but in the case in which the full aperture of the pupil was used the curve lies above the line, whereas in the previous similar diagram when a 2 millimetre diaphragm was used the curve lay below the line." The diffraction part of Mr Guild's explanation doubtless still holds. In the case of the small aperture there were visible faint diffraction rings that were not visible at full aperture.

With regard to the minimum measured width of 2 minutes, some experiments, not included in the lecture, were carried out with black objects on a white ground. One set of experiments—black circular spot on white ground—gave the following average results:

True angular diameter	Measured diameter
2′ 35″	2′ 5″
3′ 42″	3′ 15″
1′ 59″	1′ 56″

It cannot be said that the difference is appreciable. If anything, the measured diameter is less, as is to be expected if irradiation plays any part at these small angles.

The important result is that black spots on white, subtending angles less than 2 minutes are recognisable, notwithstanding the 2 minute minimum white image size. With regard to the 2 minute minimum width of slits, it should be remembered that in intense measurements of this kind, the brain may concentrate on the portion of the image that lies upon the foveola.

* See footnote, page 29.

I hope Mr Guild will not abandon his proposed investigations. The field is a very large one and its exploration would, I feel sure, yield most valuable results. The work could be conducted more satisfactorily in an establishment where the distractions are not so numerous and varied as they are in a factory.

Mr T. Smith has communicated the following note:

In his reply Mr French has disputed the accuracy of my rendering of his curves. Any error in the experimental curves is of the order of the thickness of the line by which they are shown. They are a direct tracing of Mr French's own figure, and if they are not in accord with the observations the fault lies in the original diagram from which they are derived. It is not clear however whether it is not rather curves I calculated from equation (4) which are impugned. To make sure that there is no error here Miss Gillman has recalculated for me three points on each of these curves, corresponding to $E = 60, 75, 90$. These all lie on the curves as drawn, which are therefore a correct representation of equation (4) in the cases to which they relate. As the diagrams show, the fit on the whole is less good than with the alternative formula I proposed. In deriving this formula attention was drawn to the extreme features which any exact representation of the facts may be confidently claimed to possess to make clear what the general character of the formula should be. I agree that the intermediate portions are the more important in fitting a curve to the experimental data. The illustration given was not carefully fitted, but was derived in the roughest way, yet it gave a better fit than Mr French's expression. It would not be difficult, though undeniably tedious, to fit a formula to all these curves with any assigned degree of accuracy, say, everywhere within one hundredth per cent. of the difference of illumination, or less; there would be little gained by so doing. The important fact is that Mr French's expression has properties which can at once be rejected as inconsistent with any reliable series of observations in certain regions, and does not compensate for this by any greater accuracy in the region where it can be applied. Incidentally the rejection of this formula involves the negation, as far as these observations have been recorded, of any evidence for a critical angle of about 4·2 minutes.

A

ON PROJECTION SCREENS

By CHARLES W. GAMBLE, M.Sc. (Tech.)

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Although screens for receiving projected images may be regarded as optical devices, and, reasonably, devices of a delicate nature, it is perhaps singular that there has been until recently little or no attention paid to investigating the conditions which determine their efficiency.

By far the largest use of projection screens occurs in the cinematograph theatres, which are so widely distributed. The degree of success which these places of amusement attain is largely determined by the provision which is made for the comfort of their patrons—comfort in the broadest sense of the term—and to the fulfilment of this condition the efforts of the best managements are continually devoted. Now 'comfort' in one important respect is largely conditioned by the degree to which all causes of visual fatigue or irritation are eliminated. Comfort is not alone, as many might think, the services of polite attendants, quietude, soft chairs and pleasing decorations. To secure pleasant viewing of any form of projected picture the chief thing is to utilise for the screen the most thoroughly diffusive surface, whilst economy of the energy used in the lamp house of the projector requires the employment of a surface having high reflective power. The employment of arcs of high as against medium amperage means considerable personal discomfort and inconvenience to operators from the heat (especially in view of the confinement which tends to follow the fulfilment of the rigid conditions imposed by the Local Authority as to safety) and increased risk of damage to the film under projection. Any inefficiency in the reflective capacity of the screen means a corresponding increase in the energy required in the arc to secure the necessary brightness of image and an extension in this direction involves extra cost in the working expenses of a cinematograph theatre. In consequence, attention has been given to the subject but what improvements have been made have been the results of 'practical' tests rather than quantitative methodical investigation. Perhaps the worst examples of projecting screens (indeed projection apparatus and projection generally) are to be found in the lecture rooms of Colleges devoted to the teaching of Science, where neglect of the whole subject seems to suggest that optical problems belong to that category of things existing purely in the realms of abstract ideas.

It is proposed in this paper to deal with the subject mainly in its relation to cinematography, but the observations will apply generally to ordinary static projection.

The advantage of exact information on the problem of what is the best form of projection screen and indeed on projection screens generally has recently been

supplied by a gentleman who is a member of this Society, Dr P. G. Nutting, who in an able paper records the result of some investigations into the subject ["Projection and Focussing Screens," *Trans. Ill. Eng. Soc.* 11 (1916), 92], and some little time ago when Service requirements in the Royal Air Force made it necessary to pay attention to the subject of screens for cinematography, Dr Nutting's paper proved of considerable value and frequent reference will here be made to the information which he has provided. Experiments had however been made of a comparative character before I had read the communication of which mention has been made.

It will not infrequently be found that the best optical conditions for a given projection are not always fulfilled, sometimes indeed cannot be fulfilled, and there is, moreover, a tendency on the part of operators to employ arcs far more powerful than is actually necessary. In many instances the amperage with a given size of carbons is far below what it might be. Although it has been shown that carbons will burn satisfactorily with a current density of 0·2 to 0·3 amperes per square mm. of the positive carbon it will be found that less than one-half of these amounts are taken and the reduced amperage means a less efficient crater and also what is an item of importance where there is a large consumption, the proportionately higher loss on rejected ends of the rods. Frequently, the argument in favour of the low amperage is the rather naively practical one, viz., that it saves the labour of frequently recarboning. Further, the carbons are often badly adjusted. The sum of these things (each apparently small) makes for an inefficiency which is a real loss.

The essential condition which must be fulfilled by any screen which is to receive a projected image is that the whole area must be a plane surface; strictly speaking, this means that the screen will be smooth. Broadly speaking, however, the screen may be in the ordinary way flat and yet present an appreciable 'grain' which means that it is not smooth. This quality tends to detract from its ability to render fine gradations sharply, but provided that the grain is not too pronounced it does not sensibly affect the rendering of the finer details of the picture. Some of the proprietory screens which have been offered possess this 'grain' which has even been claimed as an advantage, indeed, Dr Nutting himself has postulated as an ideal projection screen, a mirror the front face of which is composed of minute hexagonal facets each of which is a concave mirror of a particular angle. Screens in practice range from a surface of smooth white linen or cotton with a highly diffusive surface to the more or less regularly reflective surface formed by the spread of a metallic aluminium paint upon a smooth white ground.

To compare the efficiency of the chief surfaces a large frame was covered with fine white Bolton sheeting and upon this was formed a composite screen consisting of strips 6 by 1 feet arranged as follows:

(*a*) Plain white Bolton sheeting;
(*b*) Finest white calico;
(*c*) Duresco, a white composition coated on Bolton sheeting;
(*d*) Aluminium paint direct on Bolton sheeting;

(*e*) Aluminium paint on a substratum of Duresco on Bolton sheeting;
(*f*) A typical proprietory screen of the white diffusive type;
(*g*) A proprietory screen consisting of a fabric with a fine 'grained' surface with a finishing coating of aluminium paint.

This screen was then tested under strictly practical conditions, that is to say, films of different character were projected and the effects were judged by an audience who were acquainted with the practical requirements, but who were themselves unacquainted with the nature of the different surfaces. The walls of the theatre were very dark and all extraneous light was excluded. The throw was 35 feet and the size of the experimental picture projected was 4 feet 6 inches with a current of from 17–25 amperes flowing in the arc. The final judgment was that the Duresco gave the most pleasing result but that the picture brightness fell below that of the aluminium paint on Duresco as might be expected. There was a distinct falling off in brightness and in general quality in (*a*) and (*b*) but (*b*) was the better of the two. The tests which were very carefully carried out were after all only qualitative but they were strictly comparative and they dealt, moreover, with those materials which were in general use. Since it is not possible to secure and to maintain flatness by means of any form of rollable or stretched fabric and as flatness is essential for the desired result the most efficient screen, judging from these experiments, would be a specially prepared flat wall with a perfectly smooth surface properly treated with white Duresco or similar material which should, in addition, receive a final dressing of aluminium paint if the maximum luminosity be required. Actually this accords, in many instances, with the best practice. The drawback, however, to the use of aluminium as with all highly reflective surfaces, is that it limits the angle of uniform brilliancy so that observers seated at the margins of a wide theatre if they were near to the screen would be at a disadvantage as compared with those who were seated in or near the central line joining the screen and the projector. No surface painted screen, however, is permanent, particularly when exposed to the atmosphere of an ordinary theatre. Generally, the white assumes a yellow tinge and there is formed in addition a film composed of particles of dust and of tarry matter from the plentiful tobacco smoke daily produced by portions of the audience. These changes reduce the reflective power of the screen and in consequence the brilliancy of the projected picture, a small quantity of dust being very effective in this respect. On the other hand the renewal is quickly effected and at a relatively low cost and the original cost not being high the screens are altogether economical. While the reflective power of the metallic screen is high the glare due to the lack of perfect diffusivity does cause to many considerable visual discomfort, and for this reason as well as for the one given, viz., the restriction of angle of uniform illumination, they are open to real objection.

An interesting experiment was tried in which there was deposited metallic silver on the matt surface of a thin sheet of ground glass which was used as a screen, the natural surface of the glass facing the projector. So far as the formation of the image was concerned the screen was quite effective but the brilliancy was low, far lower indeed than the surfaced aluminium screen.

In his paper, Dr Nutting points out that the ideal screen should reflect all the light uniformly thrown upon it within a given angle of, say, 30 degrees from the normal and postulates that, as a measure of the 'diffusive efficiency' of any screen, the ratio of the light which is reflected within this given angle to the total light which is reflected. If this be adopted the total efficiency of some reflecting materials can be calculated from photometric data and the results of some of these determinations are reproduced from Dr Nutting's paper below, calculating the 30 degrees diffusion efficiency, total reflecting power and 30 degrees total efficiency:

Screen	30° diffusion efficiency	Reflecting power	Total efficiency
Magnesium carbonate block ...	0·25	0·87	0·22
Aluminium paint, mirroroid *B*	0·55	0·25	0·14
Becker compound	0·54	0·63	0·34
Ground mirror	0·69	0·92	0·64
Ideal screen	0·9 to 1·0	0·92	—

(*Note.* The "ideal" screen of Dr Nutting is defined as follows: a mirror, the front face of which is composed of minute hexagonal facets, each of which is a concave mirror of just sufficient diameter to refract light at a maximum angle of 15 degrees with the axis.)

The reflecting power of a screen is the ratio of the light totally reflected to the light totally incident, and the total efficiency is the ratio of the light which is reflected within the angle of 30 degrees of the normal to that which is totally incident.

Amongst the materials which were photometrically tested by Dr Nutting, some of the constants of which are given in the table above, is a glass mirror silvered on the back and then ground on the remaining or exterior surface. On reading the paper the figures appeared to be so good that it was decided to experiment at once with the material. At angles greater than 30 degrees there is with this mirror a marked falling off in the light reflected but the efficient angle is large. For the purposes of a preliminary trial a screen was made of ordinary 21 oz. sheet glass, silvered by deposition and fine sandblasted on the exterior face and this proved itself to be entirely successful. On placing this screen temporarily in contact with the ordinary fine white matt projection screen then in use, the increase in brilliancy was startling.

Mechanical considerations required that if screens of this nature were to be employed of large size, that they must be much thicker than is the case with 21 oz. glass. With 21 oz. glass the reflecting and diffusing surfaces are only about 0·1 inch apart. It did not appear correct to make the assumption that if this distance were increased, say, to 1/4 inch that the effect would be equally satisfactory but the reasonable doubt in this respect was removed by a further trial with a screen of greater thickness. It was then decided to instal a screen of reasonable dimensions for the small theatre where the main demonstrations were given and to this end one of 6 feet 6 inches by 4 feet 6 inches was made. This screen (which was mounted in a stout black wooden frame) was given a lengthy practical trial and proved itself to be, by a general consensus of opinion, entirely satisfactory. It was found that with the same film equal brilliancy resulted from the expenditure of from 1/2 to 2/3 the current consumption. There was shown to be one very striking

difference in the appearance of pictures projected on the mirror screen as compared with the ordinary opaque screen which is difficult to explain as it is difficult to demonstrate, except to those who can actually see the comparison. The pictures certainly have more of that indefinable quality 'life'—even the shadows have the quality.

An endeavour was then made to improve the effect by using a special form of obscured glass prepared by a method of fluoric acid etching. In this glass the 'surface' consists of a series of irregularly shaped very shallow cavities, the diffusivity being much less than that of ground glass. When silvered on the plane side and used as a screen with the rough surface towards the projecting lens the effect was extremely satisfactory so far as regards the brightness of image but the angle of view for uniform illumination was very small.

An objection was raised by some who had witnessed the early trials that whilst the screen would be good for exhibiting the images from films which showed normal or ordinary contrasts it would be objectionable for those where the contrasts were considerable. The high lights, it was averred, would be too bright. This seemed to be a reasonable objection, for since absolute brightness of high lights was concerned and the pleasantness or the reverse of the visual experience would be the final test, it did not seem fair to argue that the ratio of brightness of tones by which contrast is measured would be the same as for an opaque screen. It is customary with some operators under such conditions as I have stated to lower the illumination by decreasing the current flowing through the arc, a plan which is not entirely satisfactory. It suggested itself to me after some consideration that the simplest way to overcome the difficulty would be to place in front of the projecting lens a neutral tinted glass screen of low optical 'density' (density = $\log_{10}$ opacity) and this on trial was found to be perfectly effective. The effect is to dim uniformly the picture—the relation of the tones remains the same, but the absolute brightness of the high lights is lessened and this fact constitutes the effectiveness of the device. A very low density suffices and it was not found necessary to use a screen with optically worked surfaces. Actually, the quality of the screen used was about equal in flatness, and in freedom from flaws, to patent plate. This device—which appears to be of a useful character—is recommended to the attention of makers of projectors who might supply such an attachment at a low cost.

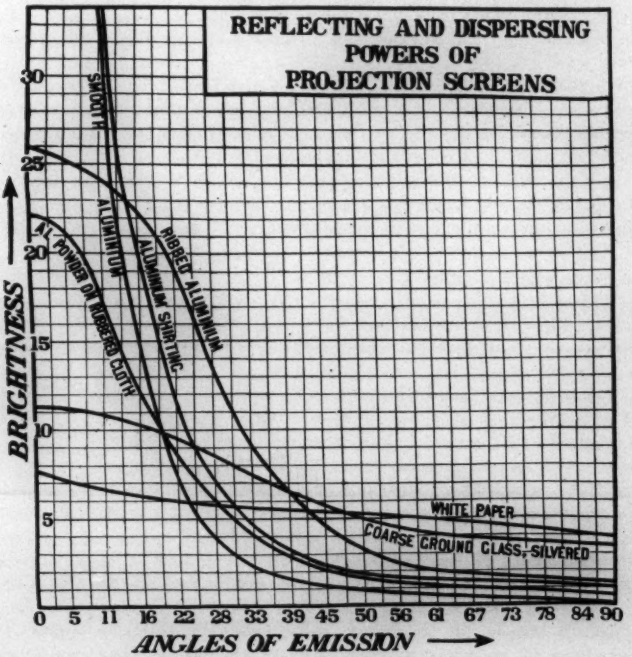
A matter of some importance when considering the general efficiency of projection screens—and this applies to all forms of screen—is the nature of the surroundings. The screen should always have a border of appreciable width and of low luminosity which condition is best secured by the use of a black fabric. This serves to isolate the projected picture from its surroundings—which is the function of any form of frame—and a device of this character is quite necessary to secure concentration of the attention of observers on the picture itself—and by its low brightness accentuates contrast between the picture and its frame. It is important also to see for these reasons (quite apart from the irritation which is caused by the reverse) that the picture fills the frame—it should not be less than the limit of the screen's size, nor should it overlap. In good cinematograph

projection this occurs; the pictures are standard and the dimensions of the screen are in this proportion. There are of course other optical and mechanical conditions which must be fulfilled in order that the projected picture may produce a pleasant sensation, such as constancy in the lamphouse and steadiness in the projector (the future projector will probably be more strongly built and mounted on a much better foundation than usually obtains now), absence of mechanical defects in the film, and uniformity in their photographic qualities. But, in the sum of the different factors which make for goodness that concerned with the screen has a high figure of value and whether it be a still or a moving picture the necessities are the same.

Whilst there are practical difficulties to be overcome in the supply of the new mirror screen formulated, of a sufficiently large size to fulfil the requirements of important picture theatres, this does not apply to many other places where projected pictures are used. This observation applies notably to Colleges and Schools where as a rule the methods and the conditions of the system of projection leave much to be desired and it is hoped that this simple communication may serve the useful purpose of drawing attention to the subject.

DISCUSSION

The President (Prof. F. J. Cheshire) drew attention to a paper on optical projection screens, by H. Lehmann, read before the German Physical Society in 1908. Some of the results obtained are illustrated in the following diagram.



The screens were illuminated with normally incident light throughout, and the intensity of the light reflected at angles to the normal, varying from zero to 90° determined with a photometer. In the diagram therefore "the abscissae represent the magnitudes of the angles of emission or angles of obliquity at which the screen picture is viewed, whilst the ordinates give the relative screen luminosities."

The author communicated the following reply.

I had not had the advantage of seeing Lehmann's paper until my attention was drawn to it by the President's remarks.

DESCRIPTION OF A GERMAN PORTABLE RANGE-FINDER

By JOHN S. ANDERSON, M.A., B.Sc., Ph.D.

(From the National Physical Laboratory)

MS. received, 3rd October, 1919. *Read, 16th October*, 1919

The author is indebted to the Director of Naval Ordnance, Admiralty, for his kind permission to exhibit a German Portable Range-finder, and to publish the following account of its mechanical and optical properties.

General Characteristics

The instrument is employed for the purpose of measuring the range of an object, of which at least one dimension is approximately known. It thus depends on the principle of utilising the object as the base. The range-finder consists essentially of a telescope, in front of the objective of which are two adjacent small-angle prisms cut in the form of discs. One of these discs is coaxial with the telescope, while the light which passes through the other is deflected by means of a step prism so as to pass into the telescopic system. Portions of this step prism are cut away to allow light which passes through the first small-angle prism to enter the main optical system. Thus in the field of view of the telescope two images of an object are formed. The separation of the images along the direction parallel to the line joining the centres of the two small-angle prisms can be varied by rotating these prisms in opposite directions about their optical axes. In order to obtain the range of an object the prisms are rotated until the two images of the object are brought into contact. Thus, if the object is a ship viewed broadside on, the bow of one image is made to touch the stern of the second image. The range of the ship and the angle which the ship subtends at the point of observation can then be read off on scales engraved on a disc which is revolved by the mechanism which rotates the prisms, provided that the length of the ship is known.

Mechanical Properties

The complete range-finder and the leather case which holds it are shown in the photograph (Fig. 1). The case bears the trade mark of Carl Zeiss, Jena. The mechanical details of the instrument are illustrated in the photograph (Fig. 2) and in the drawings (Figs. 3, 3*a*). Fig. 3 shows the instrument with the range scales removed and Fig. 3*a* is a longitudinal section of the range-finder as seen from the back. In both Figs. 3 and 3*a* the eye-piece end of the instrument is, for the sake of clearness, shown rotated through 90°. The same reference numbers are used in the drawings and the photographs.

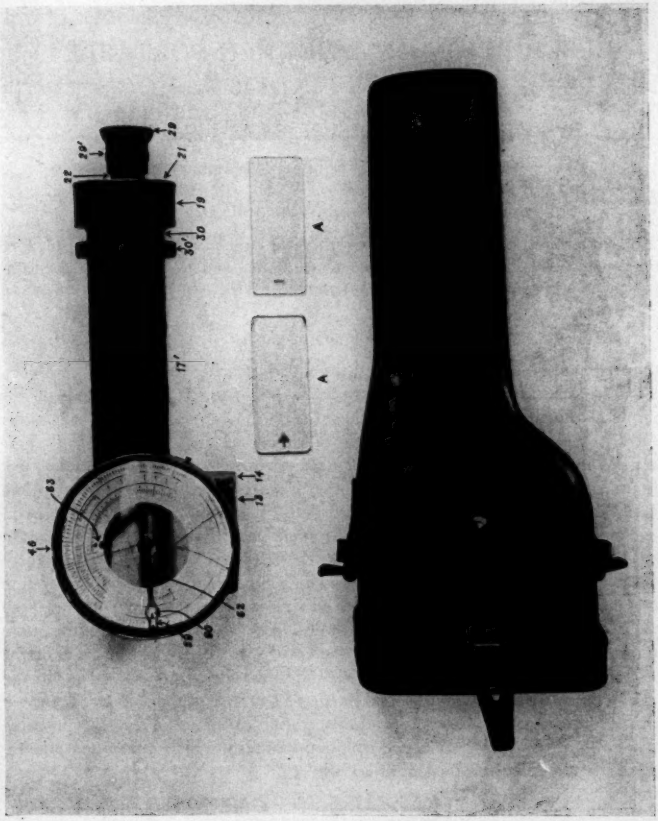


Fig. 1.

Fittings for Window and Small-angle Prisms

A rectangular cover 1 with two circular holes is provided with grooves in which a plate glass window *A* slides. The window can be held in position by a clip which fits into a groove on one edge of the window. On the outside of the cover 1 are two guide pieces 2 between which and the cover a metal screen 3 slides. This screen can be held in position by another clip 4 which fits into a groove on the edge of the screen. The cover 1 is attached to a frame 5 which has two circular holes opposite

the holes in the cover. The two small-angle prisms *B*, *B'* are held in cells 6, 6' by means of rings 7, 7' fixed to the inside of the cells; these rings bear on the prism discs at three equidistant points, and they are split at these places in order to prevent sudden shocks from damaging the glass. The prism cells 6, 6' pass through the holes in the frame 5 and are held in position by rings 8, 8'. In order to obtain the correct orientation of the prisms in their cells, the edge of each disc has a small groove which fits over a small screw projecting from the inside of each cell. These screws also serve the purpose of locking the rings 8, 8'.

Step Prism Fittings

The step prism *C* rests on a base plate 9 between guide pieces 10, 10', the plate being attached to a projection of the frame 5. The prism is pressed against the base plate by means of a metal strip 11 screwed to two pillars 12, 12' which are fixed to the same projection. A piece of cork rests between the metal strip 11 and the prism, and there is a layer of paper between the prism and the base plate.

Objective Holder

The frame 5 is fixed to one side of a rectangular box-shaped casting 13; on the opposite side a plate 14 is attached. The objective *D* is held in a cell 15 by means of a ring 16 similar to the rings 7, 7' already described. The cell is screwed into the plate 14, to the outside of which a cylindrical tube 17 is fixed. For convenience in gripping the instrument by means of this tube, it is wound round with waxed cord 17', after the manner employed in winding the handles of golf and hockey sticks, etc. There are two diaphragms 18 fixed to the inside of the tube.

Mounting for Porro Prism System

At the other end of the tube a cylindrical box 19 is fixed. A Porro prism system *E* rests between guide pieces 20 on the base of this box and is held in position by the lid 21. There is a piece of cork between the lid and the prism.

Cross-wire

A cylindrical tube 22 is screwed into the lid 21. A diaphragm 23, across which a single cross-wire 24 is stretched, is screwed into the end of the tube 22.

Eye-piece Fittings

The field lens *F* and eye lens *G* are bezelled into cells 25 and 26 which are screwed into the ends of a tube 27. This tube can move inside the tube 22, the amount and direction of motion being governed by a small guide piece 28 which fits in a spiral groove 27' cut on the outside of the tube 27; the guide is fixed to the inside of the outer tube 22. A fine screw thread is turned on the inner surface of tube 27. The purpose of this is to reduce the amount of light scattered by the surface. An eye-piece cap 29, on the outside of which a scale of dioptres is engraved, is screwed on to the rim of the cell 26 which holds the eye lens *G*. The focussing adjustment is thus made by turning the ebonite cap 29 by means of the knurled projection 29'.

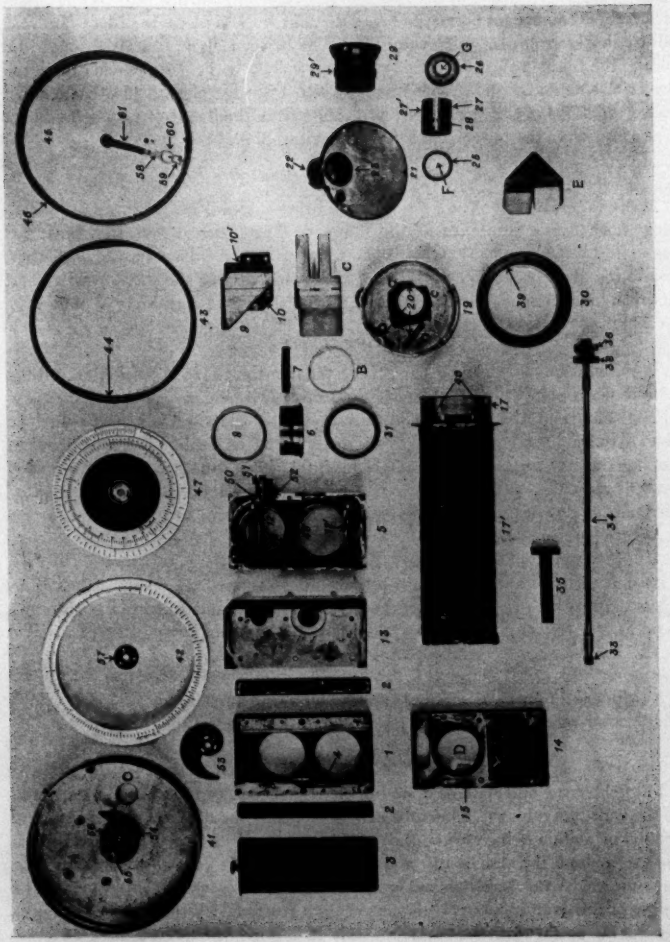


Fig. 2.

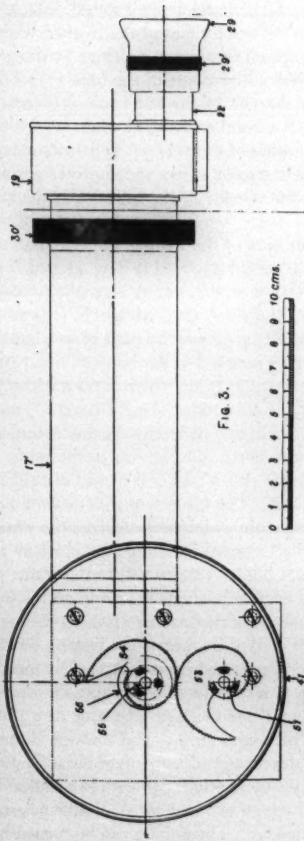


Fig. 3.

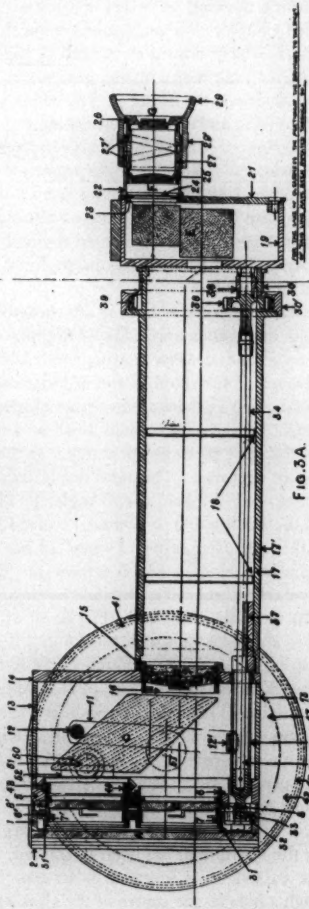


Fig. 3A.

Figs. 3, 3A.

Range-finding Mechanism

The two small-angle prisms *B*, *B'* can be made to revolve in opposite directions by turning the ring 30, which fits round the end of the tube 17, by means of the knurled rim 30'. The mechanism connected with this motion is as follows. A gear wheel 31, which is fixed to the cell 6, engages with a similar gear wheel 31', fixed to the cell 6', and with a smaller gear wheel 32 which rotates about a spindle attached to the outside of the frame 5. The wheel 32 is rotated by means of a toothed wheel 33 attached to one end of a connecting rod 34 which revolves in bearings 35 and 36. The bearing 35 is fixed to a bracket 37 screwed to the inside of the tube 17, while the bearing 36 is attached to a projection at the end of the same tube. Fixed to the other end of the connecting rod 34 there is another toothed wheel 38 which engages with a gear wheel 39 screwed to the inside of the ring 30. The rotation of the gear wheel 30 is limited to an angle of about 240°, a screw which projects from the ring 39 coming against stops 40, fixed to the outside of the tube 17, at the extremities of motion.

A flat cylindrical box 41 is attached to one side of the casting 13. An annulus 42 rests on a projecting ridge of the box and is held down by a ring 43 which is fixed by means of screws to the cylindrical portion of the box. A logarithmic scale is engraved on the annulus, and is designated "Zielhöhe" (i.e. "Height of Object"). Above the ring 43 there are two rings of black cord 44, on which a plate glass window 45 rests. This window is held down by a ring 46 screwed to the box.

In the same plane as the annulus 42 there is a disc 47 on which two logarithmic scales are engraved. The outer one is marked "Entfernung" (i.e. "Distance") and the inner one "Winkel" (i.e. "Angle"). The mechanism whereby the disc is turned when the small-angle prisms are revolved is as follows. On the side of the frame 5 next the step prism a bevel wheel 48 fits on the end of the cell 6' and is held in position by a ring 49 which screws into the cell. The correct relative orientation of bevel wheel and cell is obtained by the use of a pin which projects from the wheel and fits in a hole in the cell. The bevel wheel 48 engages in another bevel wheel 50 fixed to a spindle 51 which rotates in a bracket holder 52 attached to the frame 5. The spindle 51 projects through a hole in the bottom of the box 41, and on the top of it an excentric toothed cam 53 is fixed by means of three screws passing through slotted holes. The correct position of the cam is fixed by means of a locking screw. This cam engages in a similar excentric toothed cam 54 which is attached, by means of three screws passing through slotted holes, to a spindle 55. This spindle rotates in a bearing 56 which is screwed to the bottom of the box 41. The scale disc 47 is attached to the cam 54 by screws which pass through a ring 57 and through slotted holes in the disc. This arrangement allows of the correct adjustment of the scale disc when the instrument has been assembled. A celluloid pointer 58 with two indicator marks 59 and 60 is screwed to an arm 61, which is fixed to a spindle passing through a hole in the centre of the glass window 45. The pointer can be turned by means of a short lever 62 fixed to the spindle on the outside of the glass window.

The method of finding the range of an object when the instrument has been set so that the two images of the object are brought into contact, is as follows. The pointer 58 is turned round until the indicator mark 59 is set to the correct dimension

of the object on the fixed "Zielhöhe" scale. Then the mark 60 indicates on the "Entfernung" scale the distance of the object. At the same time the angle subtended by the object at the observer is indicated on the "Winkel" scale by means of a mark 63 painted on the under side of the window 45. The correct orientation of this window is obtained by means of a projecting screw which fits into a groove cut in the edge of the window.

Materials Employed

The screen 3, cells 6 and 6′, rings 7, 7′, 8, and 8′, guide piece 11′, pillar 12, cell 15, ring 16, guide pieces 20, tube 22, diaphragm 23, cells 25 and 26, tube 27, ring 30, gear wheels 31, 31′, and 32, bearings 35 and 36, toothed wheel 38, ring 39, stops 40, ring 43, bevel wheel 48, ring 49, bevel wheel 50, spindle 51, bracket 52, cams 53 and 54, spindle 55, bearing 56, ring 57, arm 61, and lever 62 are made of brass. The clip 4, guide piece 28, toothed wheel 33, and connecting rod 34 are steel. All the other metal parts of the range-finder are constructed of aluminium or of an aluminium alloy.

Case

A stout leather case (see Fig. 1) is provided with the instrument. There is accommodation inside the case for holding two glass windows *A* and the metal screen 3. The case has a hinged lid which is held in its closed position by a spring catch and button arrangement. Buckles, to which a strap may be attached, are provided at the sides of the case.

Optical Properties

Definition, etc.

Magnification	10
Field of view	3·7°
Definition	Not good
Colour	Considerable
Illumination of field	Poor

Details of Optical System

A. Glass Window.

Length	10	cms.
Breadth	4	,,
Thickness	·32	,,
Distance between *A* and *B*	1·0	,,

B, B′. Small-angle Prisms.

Aperture	3·2	cms.
Axial thickness	·38$_6$	,,
Distance between *B′* and entrance face of *C*	1·3	,,

The angle of each prism is approximately 1° 18′.

C. Step Prism.

Height	3·1	cms.
Breadth of entrance and exit faces	3·9	,,
Distance between exit face of *C* and *D*	·45	,,
Angle between entrance face and first reflecting surface	44° 19′	
Angle between entrance face and second reflecting surface	136° 21′	
Angle between first reflecting surface and exit face	134° 20′	
Angle between second reflecting surface and exit face	45° 0′	

The prism is mounted in such a way that the entrance and exit faces make small angles with the optical axis. The portion of the prism which is opposite the small-angle prism *B* is slotted (see Fig. 2) so as to allow light to pass direct from *B* to the objective *D*. The total height of the free portion (1·4 cm.) is less than that of the glass portion (1·7 cm.); this compensates for the loss of light in the beam which traverses the prism.

D. Objective.

Aperture	3·0	cms.
Axial thickness	·81$_5$	,,
Approximate focal length	25	,,
Distance between *D* and entrance face of *E*	17·8	,,

The objective is a cemented doublet with the flint component leading.

E. Porro Prism System.

Aperture	2·0	cms.
Height	2·2	,,
Distance between exit face of *E* and *F*	1·9	,,

The system consists of two congruent tetrahedra (sphenoids) placed in opposition and cemented together. Pieces of glass are cemented across the top and bottom of the combination for the sake of strength. The purpose of the system is to reverse the field.

Cross-wire

Width	·007$_5$ cm.

F. Field Lens.

Aperture	1·7	cms.
Axial thickness	·30	,,
Approximate focal length	4·6	,,
Distance between *F* and *G*	2·0	,,

G. Eye-lens.

Aperture	·95	cms.
Axial thickness	·37	,,
Approximate focal length	3·1	,,
Approximate focal length of combination of *F* and *G*	2·5	,,

General Remarks

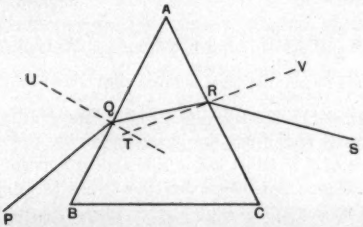
Although the instrument does not exhibit the same high-class finish as most of Zeiss's instruments, the fit of the different parts is very good. It is evidently designed for giving a rough estimate of ranges. From observations on short ranges it appears that the accuracy of setting is of the order of 1 or 2 per cent., provided that the dimensions of the object are accurately known.

The author, in conclusion, desires to express his indebtedness to Miss A. Barbara Dale, Mr A. G. Williams, M.B.E., and Mr W. J. Stockwell, of the Optics Department, for assistance in making the necessary measurements, and to Mr W. S. Walker, of the Aerodynamics Department, for the sectional drawings of the instrument.

THE INCIDENT ANGLES CORRESPONDING TO GIVEN DEVIATION PRODUCED BY A PRISM

By T. SMITH, B.A.

One of our members has suggested to me that it would be useful to have a formula giving the angle of incidence for a ray which corresponds to a given dispersion when the ray traverses a prism of known refractive index and angle. The solution to this problem is given below.



Let BAC be a principal section of the prism, A being the refracting angle, and BC being perpendicular to the bisector of A. The ray is denoted by the broken line $PQRS$, the normals at Q and R, the points of refraction, being UT and TV. When the deviation is a minimum QR is parallel to BC, the angles TQR and TRQ are each equal to half the angle A, and PQ and RS make with UT and TV angles equal to half the sum of the minimum deviation and the angle of the prism. Thus if μ is the refractive index, α is the prism angle, and δ_0 is the deviation

$$\sin \tfrac{1}{2}(\alpha + \delta_0) = \mu \sin \tfrac{1}{2}\alpha,$$

a well known equation for finding δ_0. When the deviation is not a minimum QR makes a finite angle with BC, say $\frac{\theta}{2}$, and similarly the external bisector of the angle between PQ and RS makes a finite angle $\frac{\phi}{2}$ with BC. The sum of the angles TQR and TRQ must still be equal to α, and similarly the sum of the angles PQU and VRS must be equal to the deviation δ plus α. Thus the angles may be denoted as follows:

$$PQU = \tfrac{1}{2}(\alpha + \delta + \phi), \qquad VRS = \tfrac{1}{2}(\alpha + \delta - \phi)$$
$$TQR = \tfrac{1}{2}(\alpha + \theta), \qquad QRT = \tfrac{1}{2}(\alpha - \theta),$$

and the equations of refraction are

$$\sin \tfrac{1}{2}(\alpha + \delta + \phi) = \mu \sin \tfrac{1}{2}(\alpha + \theta)$$
$$\sin \tfrac{1}{2}(\alpha + \delta - \phi) = \mu \sin \tfrac{1}{2}(\alpha - \theta)$$

or
$$\sin \tfrac{1}{2}(\alpha+\delta)\cos\tfrac{1}{2}\phi = \mu \sin\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\theta \quad \ldots.(1),$$
$$\cos\tfrac{1}{2}(\alpha+\delta)\sin\tfrac{1}{2}\phi = \mu\cos\tfrac{1}{2}\alpha\sin\tfrac{1}{2}\theta \quad \ldots.(2).$$

The problem is solved by finding ϕ in terms of α, δ and μ. Thus θ should be eliminated from the equations. This may be done by multiplying (1) throughout by $\cos\frac{1}{2}\alpha$ and squaring, and adding to (2) multiplied by $\sin\frac{1}{2}\alpha$ and squared. The result is

$$\sin^2\frac{\alpha+\delta}{2}\cos^2\frac{\alpha}{2}\cos^2\frac{\phi}{2} + \cos^2\frac{\alpha+\delta}{2}\sin^2\frac{\alpha}{2}\sin^2\frac{\phi}{2} = \mu^2\sin^2\tfrac{1}{2}\alpha\cos^2\tfrac{1}{2}\alpha,$$

and this may be rearranged in many different forms. An expression suitable for logarithmic calculation may be found by utilising the angle δ_0 in place of the refractive index in the form

$$\sin^2\frac{\phi}{2}\sin\frac{\delta}{2}\sin\left(\alpha+\frac{\delta}{2}\right) = \cos^2\frac{\alpha}{2}\sin\frac{\delta-\delta_0}{2}\sin\left(\alpha+\frac{\delta+\delta_0}{2}\right).$$

For other methods of calculation it is more convenient to use the equation

$$\cos\phi\{\cos\alpha - \cos(\alpha+\delta)\} = \mu^2\sin^2\alpha + \cos\alpha\cos(\alpha+\delta) - 1.$$

In either case, as is seen from the character of the problem, there are two solutions with values of ϕ of equal magnitude and of opposite sign, but one case is simply the other taken in the reverse direction.

Example: $\alpha = 30°$, $\delta = 20°$, $\mu = 1{\cdot}5$. $\delta_0 = 15°\ 41'$ nearly, $\phi = \pm\ 57°\ 44'$, and angle of incidence is about $53°\ 52'$ or $-\ 3°\ 52'$, as may easily be verified by tracing a ray through in the usual way.

NOTICES

The Secretary of the Department of Scientific and Industrial Research has sent the two following notices:

The Lord President of the Council has appointed Professor Joseph Ernest Petavel, D.Sc., F.R.S., M.I.Mech.E., etc., to be Director of the National Physical Laboratory in succession to Sir Richard Glazebrook, C.B., F.R.S., who retired on reaching the age limit on September 18th.

Professor Petavel is Professor of Engineering and Director of the Whitworth Laboratory in the University of Manchester. He is a member of the Advisory Committee for Aeronautics of the Air Ministry. He was educated at University College, London, and undertook scientific research at the Royal Institution and at the Davy Faraday Laboratory until 1898. He was elected John Harling Fellow of the Owens College, Manchester, in 1900 and was Scientific Manager of the Low Temperature Exhibit of the British Royal Commission for the St Louis Exhibition, 1904.

Sir Henry Alexander Miers, D.Sc., F.R.S., Vice-Chancellor of the University of Manchester, has been appointed by an Order of Council dated the 16th day of October 1919, to be a member of the Advisory Council to the Committee of the Privy Council for Scientific and Industrial Research.

SIR RICHARD GLAZEBROOK AND THE NATIONAL PHYSICAL LABORATORY

On September 18th last, Sir Richard Glazebrook, the first Director of the National Physical Laboratory, retired on reaching the age limit of 65. He took up the post on January 1st, 1900, and had thus been Director for close upon twenty years.

It was at first intended that the Laboratory should be developed from the Kew Observatory, and work was originally commenced there. Kew Observatory had then a close interest for members of the Optical Society; tests of telescopes and binoculars, sextants, photographic lenses and other optical instruments were carried out at Kew, as well as of thermometers, barometers and other meteorological instruments, the manufacture of which was mainly in the hands of opticians. There were difficulties, however, about extending the Observatory buildings, and Bushy House, Teddington, being offered for the purpose, the National Physical Laboratory was transferred to its present site, where it was opened by the present King, then Prince of Wales, in March, 1902. For some years Kew Observatory remained a Department of the Laboratory, and the test work formerly done there was continued. Under the able and energetic administration of the late Director

the development of the Laboratory at Teddington was rapid; it has grown into a great institution of first-rate national importance, carrying out tests and investigations in almost all branches of physical science and engineering, including aeronautics and ship design. It has rendered signal services in many directions during the war.

Before he became Director of the Laboratory, Dr Glazebrook was well known for his optical researches, and he has shown a constant interest in the work of the Optical Society. He was President during the session 1904–5, and again in 1911–12, and has served for many years on the Council. An Optics Division was formed at the Laboratory in 1904, to carry out work for which facilities could not be provided at Kew; at the same time photometry was taken up as part of the work of the Electrotechnics Division, and the Laboratory became the recognised authority in the country on standards of illumination. The Optics Division continued to grow, and on the transfer to Teddington in 1912 of the test work formerly carried out at Kew, it was housed in a special building, and now occupies an important position as a centre of optical research. A large number of valuable papers have been presented by the Director and his Staff to this Society and to the Physical Society; in particular, a great advance has been made in the methods of computation of optical systems.

Dr Glazebrook was knighted in 1917; he had been made a C.B. in 1910. He had held many important positions, and has served on numberless Committees, on many of which he has acted as Chairman. The assistance he is thus able to render will no doubt in the future, as in the past, be eagerly sought after; the Society desires to join with all those who know him in wishing him many years of continued activity in the aid of science and its application to industry.

F. J. S.